SECTION NAME

Date : 06/03/2024 **MOCK TEST - 7**
Time : 3 Hours 0 Minutes **Marks : 300**

Mathematics

1. The value of $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$ is

A) $\frac{3}{2}$ **B)** $\frac{2}{3}$ **C)** $\frac{3+\sqrt{3}}{2}$ **D)** $\frac{2}{3+\sqrt{3}}$

SOL:

Given $\cos^2\frac{\pi}{12}+\cos^2\frac{\pi}{4}+\cos^2\frac{5\pi}{12}$

$= 1-\sin^2\frac{\pi}{12}+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\frac{5\pi}{12}$

$= 1+\frac{1}{2}+\left(\cos^2\frac{5\pi}{12}-\sin^2\frac{\pi}{12}\right)$

$= \frac{3}{2}+\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \frac{3}{2}+\cos\frac{\pi}{2}\cos\frac{\pi}{3}$

$= \frac{3}{2}+0\cdot\frac{1}{2}=\frac{3}{2}$

2. If $z^2+z+1=0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^6+\frac{1}{z^6}\right)^2$ is

A) 54 **B)** 6 **C)** 12 **D)** 18

SOL:

$z^2+z+1=0$ we know that $1+\omega+\omega^2=0$

So $z^2=\omega^2$, $z=\omega$, $z^3=\omega^3=1$, $z^4=\omega^3\cdot\omega$

$z^5=z^3\cdot z^2=\omega^3\cdot\omega^2$ $z^6=(\omega^3)^2$ $\omega^3=1$

$\frac{1}{z}=\frac{1}{\omega}=\frac{\omega^2}{\omega^3}=\omega^2$ $\frac{1}{z^2}=\frac{1}{\omega^2}=\omega$ $\frac{1}{z^3}=1$

$\frac{1}{z^4}=\frac{1}{\omega}=\omega^2$ $\frac{1}{z^5}=\omega$ $\frac{1}{z^6}=1$

Now $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\left(z^4+\frac{1}{z^4}\right)^2+\left(z^5+\frac{1}{z^5}\right)^2+\left(z^6+\frac{1}{z^6}\right)^2$

$= (-1)^2+(-1)^2+2^2+(-1)^2+(-1)^2+(2)^2$

$= 12$

3. A box contains two white balls, three black balls and four red balls. The number of ways in which three balls can be drawn from the box if atleast one black ball is to be included in the draw, is

A) 32 **B)** 64 **C)** 128 **D)** None of these

SOL:

Box has following data.

White Balls (2)	Black balls (3)	Red balls (4)	No. of ways select 3 balls atleast 1 black
1	1	1	$^2C_1\cdot{}^3C_1\cdot{}^4C_1=24$
0	1	2	$^2C_0\cdot{}^3C_1\cdot{}^4C_2=18$
2	1	0	$^2C_2\cdot{}^3C_1\cdot{}^4C_0=3$
0	2	1	$^2C_0\cdot{}^3C_2\cdot{}^4C_1=12$
1	2	0	$^2C_1\cdot{}^3C_2\cdot{}^4C_0=6$
0	3	0	$^2C_0\cdot{}^3C_3\cdot{}^4C_0=1$

Total ways = 64

Total number of ways = 64.

4. In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, the coefficient of x^4 is

A) $\frac{405}{256}$ **B)** $\frac{504}{259}$ **C)** $\frac{450}{263}$ **D)** None of these

SOL:

Given Expansion $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$

for $(x+a)^n$ $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$.

for $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ $T_{r+1} = {}^{10}C_r \left(\frac{x}{2}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$

$T_{r+1} = {}^{10}C_r \cdot (2)^{r-10} \cdot (-3)^r\, x^{10-r} \cdot x^{-2r}$

$= {}^{10}C_r\, (-3)^r \cdot 2^{r-10} \cdot x^{10-3r}$

To get co-efficient of x^4

$x^{10-3r} = x^4$

$10-3r = 4$

$r = 2$

$\therefore T_3 = {}^{10}C_2\, (-3)^2 \cdot 2^{2-10} \cdot x^4$

$= {}^{10}C_2 \cdot 9 \cdot \frac{1}{2^8} \cdot x^4$

$= \frac{9 \cdot 10 \cdot 9}{2 \cdot 2^8} x^4 = \frac{45 \cdot 9}{2^8} x^4$

$= \frac{405}{256} x^4$

Co-efficient of $x^4 = \frac{405}{256}$.

5. If $\mathbf{1, \log_{81}(3^x+48),\ \log_9\left(3^x-\frac{8}{3}\right)}$ are in A.P, then the value of x is

A) 9 **B)** 6 **C)** 2 **D)** 4

SOL: $2\log_{81}(3^x+48) = 1+\log_9\left(3^x-\frac{8}{3}\right)$

$\Rightarrow \log_9(3^x+48) = \log_9\left[9\left(3^x-\frac{8}{3}\right)\right]$

$\Rightarrow 3^x+48 = 9.3^x-24$

$\Rightarrow 8\,.\,3^x = 72$

$\Rightarrow 3^x = 9$

$\Rightarrow x = 2$

6. The point (4, 1) undergoes the following three successive transformations
(A) Reflection about the line y = x – 1
(B) Translation through a distance 1 unit along the positive *x*-axis
(C) Rotation through an angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.
Then, the coordinates of the final point are

A) (4, 3) **B)** $\left(\frac{7}{2},\frac{7}{2}\right)$ **C)** $(0, 3\sqrt{2})$ **D)** (3, 4)

SOL:

P(4,1)

(A) Reflection about $x-y-1=0$

$\frac{x-4}{1} = \frac{y-1}{-1} = \frac{-2(4-1-1)}{2}$

$x-4 = -2$, $y-1 = 2$

$x = 2$ $y = 3$

A (2, 3)

Translation through 1 unit along positive x-axis

$\therefore$ B (3,3)

Rotation through angle $\frac{\pi}{4}$ in Anti clockwise direction.

$\therefore \theta = \frac{-\pi}{4}$

$\therefore x = 3\cos\frac{\pi}{4} - 3\cos\frac{\pi}{4} = 0$

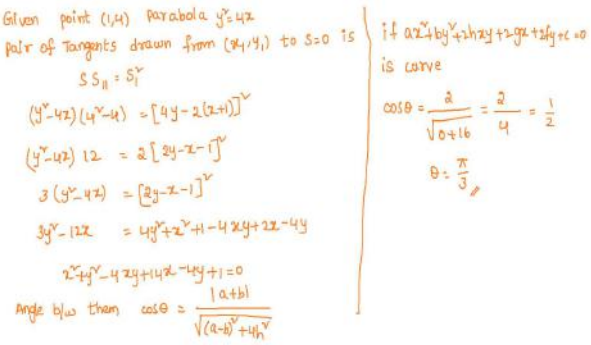
$\therefore y = 3\cos\frac{\pi}{4} + 3\cos\frac{\pi}{4} = 3\sqrt{2}$

$\therefore (0, 3\sqrt{2})$ is the final point.

7. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

A) $\pi/6$ **B)** $\pi/4$ **C)** $\pi/3$ **D)** $\pi/2$

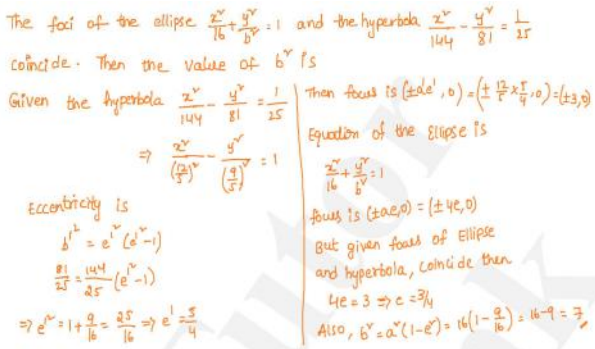
SOL:



8. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is

A) 1 **B)** 5 **C)** 7 **D)** 9

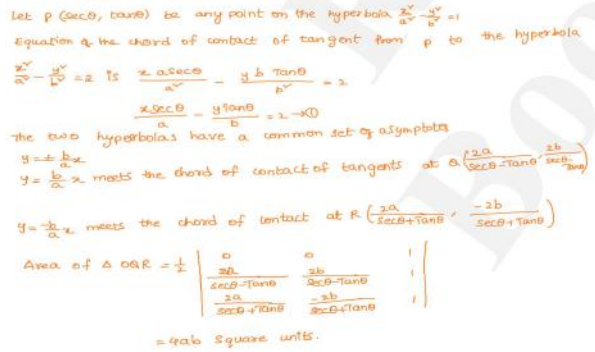
SOL:



9. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. The area cut off by the chord of contact on the asymptotes is equal to

A) $\frac{ab}{2}$ **B)** ab **C)** $2ab$ **D)** $4ab$

SOL:



10. Let R be a relation on a set A such that $R=R^{-1}$, then R is

A) Reflexive **B)** Symmetric **C)** Transitive **D)** None of these

SOL:

Given $R=R^{-1}$

Let us consider a relation such that $R=R^{-1}$

$R=\{(1,2),(2,3),(3,3),(4,2),(3,2),(3,2),(2,4),(2,1)\}$

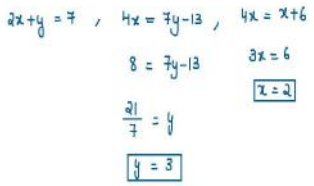
We can observe that $(x,y)\in R\rightarrow(y,x)\in R$

'R' is symmetric.

11. $[2x+y4x5x-74x]\begin{bmatrix}2x+y & 4x\\ 5x-7 & 4x\end{bmatrix}=[77y-13yx+6]\begin{bmatrix}7 & 7y-13\\ y & x+6\end{bmatrix}$, then the value of $x+y$ is

A) $x=3, y=1$ **B)** $x=2, y=3$ **C)** $x=2, y=4$ **D)** $x=3, y=3$

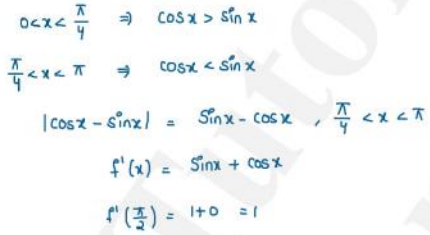
SOL:



12. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{2}\right)$ is equal to

A) 1 **B)** −1 **C)** 0 **D)** 2

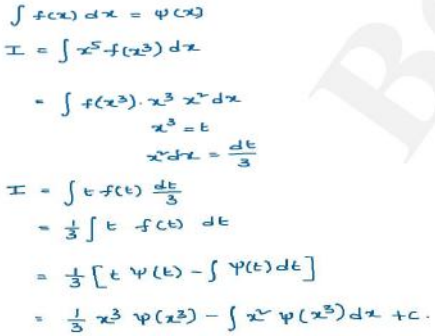
SOL:



13. If $\int f(x)\,dx = \psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to

A) $\frac{1}{3}[x^3\psi(x^3)-\int x^2\psi(x^3)dx]+C$ **B)** $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$ **C)** $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$ **D)** $\frac{1}{3}[x^3\psi(x^3)-\int x^3\psi(x^3)dx]+C$

SOL:



14. The value of $\int_0^{2\pi}\frac{\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}\,dx$ is

A) $\pi/2$ **B)** π **C)** 2π **D)** 4π

SOL:

Here $f(x)=\frac{\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}$ for which $f(x)=f(2\pi-x)$

$\Rightarrow I=\int_0^{2\pi}\frac{\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}dx$

$=2\int_0^{\pi}\frac{\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}dx$

Again $f(x)=f(\pi-x)$

$\Rightarrow I=4\int_0^{\pi/2}\frac{\cos^{2n}x}{\cos^{2n}x+\sin^{2n}x}dx \rightarrow (1)$

$=4\int_0^{\pi/2}\frac{\cos^{2n}\left(\frac{\pi}{2}-x\right)}{\cos^{2n}\left(\frac{\pi}{2}-x\right)+\sin^{2n}\left(\frac{\pi}{2}-x\right)}dx$

(by property IV)

$=4\int_0^{\pi/2}\frac{\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}dx \rightarrow (2)$

Adding (1) & (2) we have $2I=4\int_0^{\pi/2}1\,dx$

$\Rightarrow I=\pi$

15. The area bounded by the circle $x^2 + y^2 = 8$, the parabola $x^2 = 2y$ and the line $y = x$ in $y \geq 0$ is

A) $\frac{2}{3}+2\pi$ **B)** $\frac{2}{3}-2\pi$ **C)** $\frac{2}{3}+\pi$ **D)** $\frac{2}{3}-\pi$

SOL:

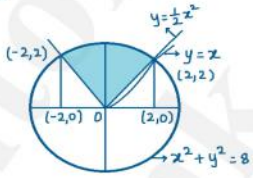
Required area

$=\int_{-2}^{2}\sqrt{8-x^2}\,dx-\int_{-2}^{0}\frac{1}{2}x^2\,dx-\int_0^2 x\,dx$

$=2\left[\frac{x}{2}\sqrt{8-x^2}+\frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2-\frac{4}{3}-2$

$=2\left[2+4\cdot\frac{\pi}{4}\right]-\frac{10}{3}$

$=\frac{2}{3}+2\pi$



16. The solution of $x^3\cdot\frac{dy}{dx}+4x^2\tan y=e^x.\sec y$ satisfying x = 1; y = 0 is

A) $\tan y = (x-2)e^x \log x$ **B)** $\sin y = e^x (x-1) x^{-4}$ **C)** $\tan y = (x-1) e^x x^{-3}$ **D)** $\sin y = e^x (x-1) x^{-3}$

SOL: We have $\cos y\cdot\frac{dy}{dx}+\frac{4}{x}\sin y=\frac{e^x}{x^3}$

Let $\sin y = t \Rightarrow \cos y\cdot\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+\frac{4}{x}t=\frac{e^x}{x^3}$

$I.F=e^{\int\frac{4}{x}dx}=x^4$

$\therefore$ The solution is $t.x^4=\int x^4.\frac{e^x}{x^3}dx$

$\sin y \cdot x^4 = x.e^x - e^x + c$

Since x = 1 and y = 0 c is also '0'

$\therefore \sin y = e^x (x-1)x^{-4}$

17. **a, b, c** are three vectors, such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, $|\mathbf{a}|=1, |\mathbf{b}|=2, |\mathbf{c}|=3$, then $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}$ is equal to

A) 0 **B)** −7 **C)** 7 **D)** 1

SOL:

$\bar{a}+\bar{b}+\bar{c}=0$

Squaring on both sides

$|\bar{a}+\bar{b}+\bar{c}|^2=0$

$|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})=0$

$(\because$ given $|a|=1,\ |b|=2,\ |c|=3)$

$1^2+2^2+3^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})=0$

$1+4+9+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})=0$

$\therefore(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})=\frac{-14}{2}$

$\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a}=-7.$

18. The angle between two planes $2x - y + z = 6$ and $x + 2y + 3z = 3$ is

A) $\cos^{-1}\left(\frac{1}{2}\sqrt{1/7}\right)$ **B)** $\cos^{-1}\left(\frac{1}{2}\sqrt{2/7}\right)$ **C)** $\cos^{-1}\left(\frac{1}{2}\sqrt{3/7}\right)$ **D)** $\cos^{-1}\left(\frac{1}{2}\sqrt{4/7}\right)$

SOL:

Given the two planes $2x-y+z=6,\ x+2y+3z=3$

$\therefore P_1=2x-y+z=6$

$P_2=x+2y+3z=3$

$\therefore\cos\theta=\frac{2+(-2)+3}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+2^2+3^2}}$

$=\frac{3}{\sqrt{6}\sqrt{14}}$

$=\sqrt{\frac{3}{28}}$

$=\frac{1}{2}\sqrt{\frac{3}{7}}\Rightarrow\theta=\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)$

19. *A* speaks truth in 60% cases and *B* speaks truth in 70% cases. The probability that they will say the same thing while describing a single event is

A) 0.56 **B)** 0.54 **C)** 0.38 **D)** 0.94

SOL:

Given A speaks 60% cases truth.

$P(A)$ = probability of A speaks truth $=\frac{60}{100}$

B speaks 70% cases truth.

$P(B)$ = probability of B speaks truth $=\frac{70}{100}$

$P(\bar{A})=1-P(A)=1-\frac{60}{100}=\frac{40}{100}$ = probability 'A' speak false.

$P(\bar{B})=1-P(B)=1-\frac{70}{100}=\frac{30}{100}$ = probability 'B' speak false

$\therefore$ The probability that A and B say the same thing while describing single event is $=P(A)P(B)+P(\bar{A})\cdot P(\bar{B})$

$=\frac{60}{100}\cdot\frac{70}{100}+\frac{40}{100}\cdot\frac{30}{100}=\frac{42}{100}+\frac{12}{100}=\frac{54}{100}=0.54$

20. The minors of -4 and 9 and the co-factors of -4 and 9 in determinant $\begin{vmatrix}-1&-2&3\\-4&-5&-6\\-7&8&9\end{vmatrix}$ are respectively

A) $42,3;-42,3$ **B)** $-42,-3;42,-3$ **C)** $42,3;-42,-3$ **D)** $42,3;42,3$

SOL: Minor of $-4=\begin{vmatrix}-2&3\\8&9\end{vmatrix}=-42, 9=\begin{vmatrix}-1&-2\\-4&-5\end{vmatrix}=-3$

and cofactor of $-4=(-1)^{2+1}(-42)=42,$

cofactor of $9=(-1)^{3+3}(-3)=-3.$

21. If the total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B, then the number of elements in the set A is

SOL:

$2^{n(A)} = 2^{n(B)} + 56$

$\Rightarrow 2^{n(A)} - 2^{n(B)} = 56 = 64 - 8$

$\Rightarrow 2^{n(A)} - 2^{n(B)} = 2^6 - 2^3$

$\Rightarrow n(A) = 6$.

22. If $\cos A = \frac{3}{4}$, then $32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right) =$

SOL:

Given $\cos A = \frac{3}{4}$

$32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right)$

$= 16\left(2 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right)\right)$

$= 16\left[\cos\left(\frac{5A}{2} - \frac{A}{2}\right) - \cos\left(\frac{A}{2} + \frac{5A}{2}\right)\right]$

$= 16\left[\cos 2A - \cos(3A)\right]$

$= 16\left\{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\right\}$

$= 16\left\{\left(2 \cdot \frac{9}{16} - 1\right) - \left(4 \cdot \frac{27}{64} - 3 \cdot \frac{3}{4}\right)\right\}$

$= 16\left\{\left(\frac{9}{8} - 1\right) - \left(\frac{27}{16} - \frac{9}{4}\right)\right\}$

$= 16\left\{\frac{1}{8} - \left(-\frac{9}{16}\right)\right\}$

$= 16\left(\frac{1}{8} + \frac{9}{16}\right)$

$= 16\left(\frac{11}{16}\right) = 11$

$\therefore 32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right) = 11$

23. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is equal to _____

SOL: We have $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$

$= \left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^8 + \left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]^8$

$= \cos 2\pi + i \sin 2\pi + \cos 2\pi - i \sin 2\pi$

$= 2 \cos 2\pi$

$= 2$ (By de-moivre's theorem)

24. $\lim_{x \to 0} \frac{\log(1+x+x^2) + \log(1-x+x^2)}{\sec x - \cos x}$ is equal to

SOL:

$\lim_{x \to 0} \frac{\log(1+x+x^2) - \log(1-x+x^2)}{\sec x - \cos x}$

$= \lim_{x \to 0} \frac{\log\left[(1+x^2)^2 - x^2\right]}{\left(\frac{1-\cos^2 x}{\cos x}\right)}$

$= \lim_{x \to 0} \frac{\log\left(1+x^2(1+x^2)\right)}{x^2(1+x^2)} \cdot \frac{x^2(1+x^2)}{\frac{\sin x}{x} \cdot \frac{\tan x}{x} \cdot x^2} = 1$

25. The A.M. of a set of 50 numbers is 38. If two numbers of the set, namely 55 and 45 are discarded, the A.M. of the remaining set of numbers is

SOL:

$$\frac{50\times 38-(55+45)}{48} = \frac{50\times 38-100}{48}$$

$$= \frac{50(38-2)}{48}$$

$$= \frac{50\times 36}{48} = \frac{75}{2}$$

$$= 37.5$$

26. The number of solutions to the equation $\tan^{-1}\left(\frac{x}{3}\right)+\tan^{-1}\left(\frac{x}{2}\right)=\tan^{-1}x$ is

SOL:

$\tan^{-1}\left(\frac{x}{3}\right)+\tan^{-1}\left(\frac{x}{2}\right)=\tan^{-1}x$

$\tan^{-1}\left[\frac{x/3+x/2}{1-x^2/6}\right]=\tan^{-1}x$

where $x>0$ and $\frac{x^2}{6}<1 \Rightarrow -\sqrt{6}<x<\sqrt{6}$

Now, $\left(\frac{5x}{6-x^2}\right)=x$

$\Rightarrow x=0$, or $x^2-1=0 \Rightarrow x=\pm 1$,

Therefore, $x=\{-1,0,1\}$

$\therefore$ 3 solution

27. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t additional number of workers x is given by $\frac{dp}{dx}=100-12\sqrt{x}$. If the firm employees 25 more workers, then the new level of production of items is

SOL:

$$\frac{dp}{dx}=100-12\sqrt{x}$$

$$P(x)=100x-\frac{12(x)^{3/2}}{3/2}+c$$

$x=0,\ P(x)=2000$

$\Rightarrow c=2000$

$x=25$

$$P(x)=100(25)-12\left(\frac{2}{3}\right)(25)^{3/2}+2000$$

$$P(x)=3500$$

28. If the vectors $\alpha\hat{i}+\hat{j}+\hat{k},\ \hat{i}+\beta\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+\gamma\hat{k}$ $\alpha, \beta, \gamma \neq 1$ are coplanar, then the value of $\frac{1}{1-\alpha}+\frac{1}{1-\beta}+\frac{1}{1-\gamma}$ is

SOL:

$(\vec{a}\ \vec{b}\ \vec{c})=0$

$$\Rightarrow \begin{vmatrix}\alpha & 1 & 1\\ 1 & \beta & 1\\ 1 & 1 & \gamma\end{vmatrix}=0$$

$$\Rightarrow \begin{matrix}c_1\to c_1-c_2\\ c_2\to c_2-c_3\end{matrix}\begin{vmatrix}\alpha-1 & 0 & 1\\ 1-\beta & \beta-1 & 1\\ 0 & 1-\gamma & \gamma\end{vmatrix}=0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\begin{vmatrix}-1 & 0 & \frac{1}{1-\alpha}\\ 1 & -1 & \frac{1}{1-\beta}\\ 0 & 1 & \frac{\gamma}{1-\gamma}\end{vmatrix}=0$$

$$\Rightarrow \frac{1}{1-\alpha}(1)-\frac{1}{1-\beta}(-1)+\frac{\gamma}{1-\gamma}(1)=0$$

$$\Rightarrow \frac{1}{1-\alpha}+\frac{1}{1-\beta}+\frac{\gamma}{1-\gamma}=0$$

$$\Rightarrow \frac{1}{1-\alpha}+\frac{1}{1-\beta}+\frac{1-(1-\gamma)}{1-\gamma}=0$$

$$\Rightarrow \frac{1}{1-\alpha}+\frac{1}{1-\beta}+\frac{1}{1-\gamma}=1$$

29. If the origin is the centroid of a ΔABC having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, then a =___

SOL:

Centroid = (0, 0, 0)

$$0 = \frac{x_1 + x_2 + x_3}{3}$$

$$\Rightarrow \frac{a + (-2) + 4}{3} = 0$$

$$\Rightarrow a = -2$$

30. A die is thrown. Let *A* be the event that the number obtained is greater than 3. Let *B* be the event that the number obtained is less than 5. Then, *P(A∪B)* is

SOL:

A - greater than 3 {4, 5, 6}

P(A) = 3/6

B - less than 5 {1, 2, 3, 4}

P(B) = 4/6

P(A∩B) = 1/6 {4}

P(A∪B) = P(A) + P(B) − P(A∩B)

$= \frac{7}{6} - \frac{1}{6} = \frac{6}{6} = 1.$

Physics

31. The velocity v (in cm/sec) of a particle is given in terms of time t (in sec) by the relation $v = at + \frac{b}{t+c}$; the dimensions of a, b and c are

A) a = L^2, b = T, c = LT^2 **B)** a = LT^2, b = LT, c = L **C)** a = LT^{-2}, b = L, c = T **D)** a = L, b = LT, c = T^2

SOL:

$$V = at + \frac{b}{t+c}$$

By principle of Homogeneity

V = at ⇒ a = ms^{-2} → $M^0L^1T^{-2}$

D.F of t = D.F of c c = $M^0L^0T^1$

D.F of V = D.F of $\frac{b}{t+c}$

$M^0L^1T^{-1} = \frac{b}{T^1}$ ⇒ b = $M^0L^1T^0$

32. An aeroplane is rising vertically with acceleration *f*. Two stones are dropped from it at an interval of time *t*. The distance between them at time *t′* after the second stone is dropped will be

A) $\frac{1}{2}(g+f)\,tt'$ **B)** $\frac{1}{2}(g+f)(t+2t')t$ **C)** $\frac{1}{2}(g+f)(t-t')^2$ **D)** $\frac{1}{2}(g+f)(t+t')^2$

SOL:

The displacement between first stone and aeroplane after 't' seconds

$S_1 = \frac{1}{2}(g+f)t^2$ [∵ u = 0, $S = ut + \frac{1}{2}at^2$]

Velocity of aeroplane = u + ft

Velocity of first stone = u − gt

u → velocity of aeroplane when first stone was dropped.

The velocity of second stone = $V_{aeroplane} - V_{first\ stone}$

$= u + ft - u + gt = (g+f)t$

⇒ The relative displacement between first and second stone after 't'.

$S_2 = (g+f)t \times t'$

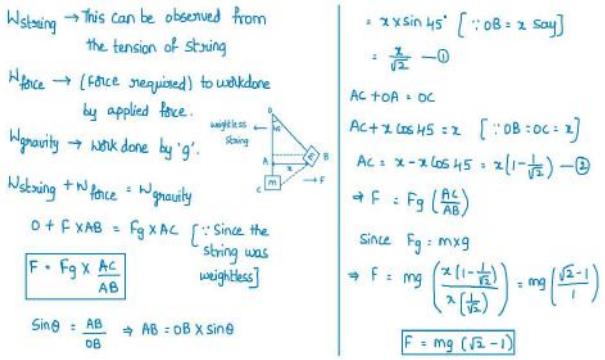
$\Rightarrow S_1 + S_2 = \frac{1}{2}(g+f)t^2 + (g+f)(t \times t')$

$= (g+f) \times \frac{1}{2}(t + 2t')t$

33. A mass of *M* kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

A) $Mg\left(\sqrt{2}+1\right)$ B) $Mg\sqrt{2}$ C) $\frac{Mg}{\sqrt{2}}$ D) $Mg\left(\sqrt{2}-1\right)$

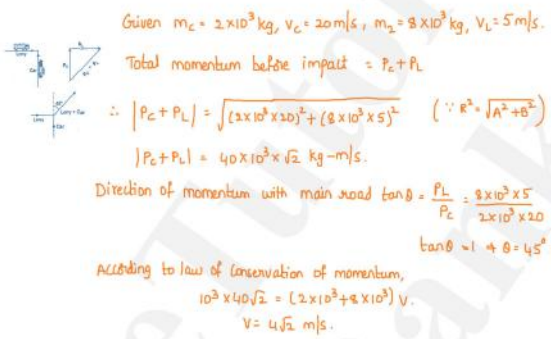
SOL:



34. A car weighing 2×10^3 kg and moving 20 m/s along a main road collides with a lorry of mass 8×10^3 kg which emerges at 5 m/s from a cross road at right angles to the main road. If the two vehicles lock, what will be their velocity after the collision?

A) $4/\sqrt{2}$ m/s, 45° with cross road B) $4/\sqrt{2}$ m/s, 60° with cross road C) $4/\sqrt{2}$ m/s, 60° with main road D) $4/\sqrt{2}$ m/s, 45° with main road

SOL:



35. The specific heat at constant volume for the monatomic argon is 0.075 kcal/kg-K, whereas its gram molecular specific heat is C_V = 2.98 cal/mol/K. The mass of the argon atom is (Avogadro's number = 6.02×10^{23} molecules/mol)

A) 6.60×10^{-23} g B) 3.30×10^{-23} g C) 2.20×10^{-23} g D) 13.20×10^{-23} g

SOL:

Molar specific heat = Molecular weight × gram specific heat

$C_V = M \times c_V$

2.98 cal/mol-K $= M \times 0.075$ kcal/kg-K

$= M \times \frac{0.075 \times 10^3}{10^3}$ cal/g-K

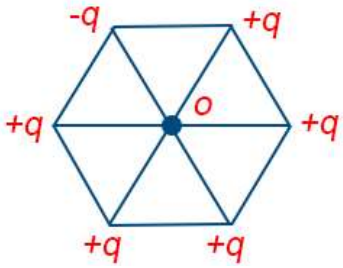
∴ Molecular weight of argon

$M = \frac{2.98}{0.075} = 39.7$ g

i.e., mass of 6.023×10^{23} atom = 39.7 g

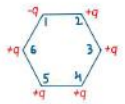
Therefore, mass of single atom $= \frac{39.7}{6.023 \times 10^{23}} = 6.60 \times 10^{-23}$ g

36. Six point charges are arranged at the vertices of regular hexagon of side length *a* (shown in Figure). The magnitude of electric field at the centre of regular hexagon is



A) $\frac{q}{4\pi\varepsilon_0 a^2}$ **B)** Zero **C)** $\frac{q}{2\pi\varepsilon_0 a^2}$ **D)** None of these

SOL:



Fields due to charges at 2, 3, 5 and 6 gets cancelled.

Field due to 1 and 4 are added

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$$

$$= \frac{1}{2\pi\varepsilon_0}\frac{q}{a^2}$$

37. Two equal negative charge $-q$ are fixed at the fixed points $(0, \alpha)$ and $(0, -\alpha)$ on the Y-axis. A positive charge Q is released from rest at the point $(2\alpha, 0)$ on the X-axis. The charge Q will

A) Execute simple harmonic motion about the origin **B)** Move to the origin and remain at rest **C)** Move to infinity

D) Execute oscillatory but not simple harmonic motion

SOL:

Say positive charge is on x-axis,

force is given by $F = \frac{-qQ}{4\pi\varepsilon_0(a^2+x^2)}$

because of symmetry only $\perp^{lar}$ component exists

$$F' = F\cos\theta = \frac{-qQ}{4\pi\varepsilon_0(a^2+x^2)}\left[\frac{x}{\sqrt{a^2+x^2}}\right]$$

$$= \frac{-qQx}{4\pi\varepsilon_0(a^2+x^2)^{\frac{1}{2}+1}}$$

$$= \frac{-qQx}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}$$

force due to two charge

$$2F' = \frac{-2qQx}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}$$

force is not exactly proportional to x, hence not exactly S.H.M and motion will be oscillatory.

38. A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of wire Q is 0.075 Nm^{-1} and carries a current of 25 A. Find the positive of wire Q from P so that wire Q remains suspended due to the magnetic repulsion. Also indicate the direction of current in Q with respect to P.

A) $\frac{1}{2} \times 10^{-2}$m **B)** $\frac{1}{3} \times 10^{-2}$m **C)** $\frac{1}{4} \times 10^{-2}$m **D)** $\frac{1}{5} \times 10^{-2}$m

SOL:

As force per unit length between two parallel current carrying wires separated by a distance d is given by

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d}$$

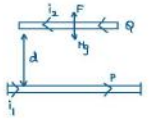
and is repulsive if the current in the wires is in opposite direction. So, inorder that wire Q may remain suspended, the force F on it must be repulsive and equal to its weight i.e, the current in the two wires must be in opposite directions and

$$F = Mg, \text{ i.e., } \frac{F}{L} = \frac{Mg}{L}$$

$$\frac{\mu_0}{4\pi}\frac{2i_1i_2}{d} = \frac{Mg}{L} \left[\text{as } \frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{d}\right]$$

$$\Rightarrow d = 10^{-7} \times \frac{2\times 50\times 25}{0.075} = \frac{1}{3}\times 10^{-2}\text{ m}$$

$$\left[\text{as } \frac{Mg}{L} = 0.075\ Nm^{-1}\right]$$



39. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio H/F, When F is field due to magnet will be

A) 3 **B)** 1/3 **C)** $\sqrt{3}$ **D)** $1/\sqrt{3}$

SOL:

$$\frac{T_2}{T_1} = \sqrt{\frac{H}{H+F}} = \frac{1}{2}$$

$$\frac{H+F}{H} = \frac{4}{1}$$

$$H+F = 4H$$

$$F = 3H \Rightarrow \frac{H}{F} = \frac{1}{3}$$

40. The self inductance of a choke coil is 10 *mH*. When it is connected with a 10V dc source, then the loss of power is 20 *watt*. When it is connected with 10 *volt* ac source loss of power is 10 *watt*. The frequency of ac source will be

A) 50 *Hz* **B)** 60 *Hz* **C)** 80 *Hz* **D)** 100 *Hz*

SOL:

From theory we have, $P = \frac{V^2}{R}$ (with dc)

$$R = \frac{V^2}{P}$$

Given $V = 10V$, $P = 20$ watt, $L = 10mH$.

$$\therefore R = \frac{10 \times 10}{20} = 5\Omega$$

Also, $P = \frac{V_{rms}^2 R}{Z^2}$ (with ac)

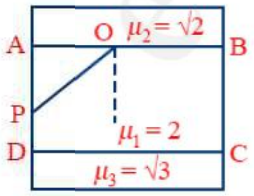
$$Z^2 = \frac{V_{rms}^2 \times R}{P} = \frac{10 \times 10 \times 5}{10} \quad (\text{Given } P = 10)$$

$$Z^2 = 50\Omega^2$$

Also, $Z^2 = R^2 + 4\pi^2 v^2 L^2$

$$50 = (5)^2 + 4(3.14)^2 v^2 (10 \times 10^{-3})^2 \Rightarrow v = 80\,Hz.$$

41. A parallel sides slab ABCD of refractive index 2 is sand witch between two slabs of refractive indices $\sqrt{2}$ and $\sqrt{3}$ as shown in the figure. The minimum value of angle θ such that the ray PQ suffers total internal reflection at both the surfaces AB and CD is



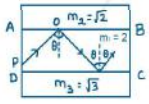
A) 30° **B)** 45° **C)** 60° **D)** 75°

SOL:

For total internal reflection at surface AB, angle θ must be greater than or equal to the critical angle i_1 given by

$$\sin i_2 = \frac{\mu_2}{\mu_1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

which gives $i_1 = 45°$



For total internal reflection at surface CD, angle θ must be greater than or equal to the critical angle i_2 given by

$$\sin i_2 = \frac{\mu_3}{\mu_1} = \frac{\sqrt{3}}{2}$$

which gives $i_2 = 60°$

Hence, for total internal reflection at both the surfaces AB and CD, the minimum value of $\theta = 60°$

42. A nucleus with Z = 92 emits the following in a sequence
α, α, β^-, β^-, α, α, α, α; β^-, β^-, α, β^+, β^+, α.
The Z of the resulting nucleus is

A) 76 **B)** 78 **C)** 82 **D)** 74

SOL:

A nucleus with $Z = 92$

Decrease in $Z = 8 \times 2 = 16$

4 $\longrightarrow \beta^-$ particles are emitted $4\left(-1\beta^0\right)$

$\Rightarrow$ Increase in $Z = 4 \times 1 = 4$

2 $\longrightarrow \beta^+$ particles are emitted $2\left(1\beta^0\right)$

$\Rightarrow$ Decrease in $Z = 2 \times 1 = 2$

$\Rightarrow$ $Z_R = 92 - 16 + 4 - 2$

$Z_R = 78$

43. A Carnot engine whose sink is at 300K has an efficiency of 40%. By how much should the temperature of source be increased so as to increase its efficiency by 50% of original efficiency?

A) 275 K **B)** 325 K **C)** 250 K **D)** 380 K

SOL:

We know that efficiency of carnot engine $(\eta) = 1 - \frac{T_2}{T_1}$

Here, T_1 is the temperature of source and T_2 is temperature of sink.

Given $\eta = 40\% \Rightarrow \frac{40}{100} = 0.4$

$T_2 = 300$ K

$\therefore 0.4 = 1 - \frac{300}{T_1}$

$T_1 = \frac{300}{1-0.4} = 500$ K

Let the temperature of source is increased by x then efficiency becomes

$\eta = 40\% + 50\%$ of η

$\eta' = \frac{40}{100} + \frac{50}{100} \times 0.4$

$\eta' = 0.4 + 0.5 \times 0.4 = 0.6$

Hence, $0.6 = 1 - \frac{300}{500+x}$

$\frac{300}{500+x} = 0.4 \Rightarrow x = 750 - 500$

$x = 250$ K

44. Huygens' wave theory allows us to know

A) the wavelength of the wave **B)** the velocity of the wave **C)** the amplitude of the wave **D)** the propagation of wavefronts

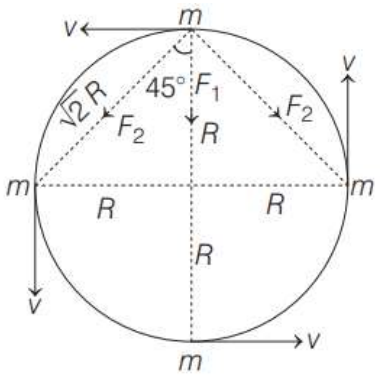
SOL:

Huygen's wave theory allows us to know the propagation of wavefronts.

45. Four identical particles of equal masses $\mathbf{1\ kg}$ made to move along the circumference of a circle of radius $\mathbf{1\ m}$ under the action of their own mutual gravitational attraction. The speed of each particle will be

A) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$ **B)** $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$ **C)** $\sqrt{G(1+2\sqrt{2})}$ **D)** $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

SOL:



Given, $m = 1\ \text{kg}, R = 1\ \text{m}$

We know that,

$F = \frac{Gm_1m_2}{r^2}$

$\because \quad F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$

and $\quad F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$

Net force on one particle,

$F_{\text{net}} = F_1 + F_2\cos 45^\circ + F_2\cos 45^\circ$

$= F_1 + 2F_2\cos 45^\circ$

$= \frac{Gm^2}{4R^2} + 2\left(\frac{Gm^2}{2R^2}\right)\cdot\frac{1}{\sqrt{2}}$

$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{\sqrt{2}R^2}$

$= \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$

As the gravitational force provides the necessary centripetal force, so

$F_{\text{net}} = F_c = \frac{mv^2}{R}$

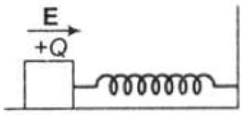
Here, F_c = centripetal force.

$\Rightarrow \frac{Gm^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = \frac{mv^2}{R}$

$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$

$\Rightarrow \quad v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$

46. A wooden block performs SHM on a frictionless surface with frequency v_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field E is switched on as shown, then the SHM of the block will be



A) Of the same frequency and with shifted mean position **B)** Of the same frequency and with the same mean position

C) Of changed frequency and with shifted mean position **D)** Of changed frequency and with the same mean position

SOL: Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position.

For example, in the given equation mean position is at natural length of spring in the absence of electric field. Whereas in the presence of electric field mean position will be obtained after a compression of x_0.

Where x_0 is given by

$kx_0 = QE$

$x_0 = \frac{QE}{k}$

47. The electric field of a plane electromagnetic wave varies with time of amplitude 2Vm^{-1} propagating along z-axis. The average energy density of the magnetic field is (in Jm^{-3})

A) 13.29×10^{-12} **B)** 8.85×10^{-12} **C)** 17.72×10^{-12} **D)** 4.43×10^{-12} **E)** 2.22×10^{-12}

SOL: $U = \frac{1}{2} \times \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} \times \frac{1}{2} \times 8.85 \times 10^{-12} \times (2)^2$

$= 8.85 \times 10^{-12}\text{Jm}^{-3}$

48. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

A) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ **B)** $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ **C)** $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ **D)** $\alpha = 25.00\pi, \beta = \pi$

SOL: Given : λ = 0.08 m T = 2.0 s

The given equation is :

y(x, t) = 0.005 cos (αx – βt)

On comparing with standard equation:

y(x, t) = A cos(kx – wt)

$\therefore \alpha = k = \frac{2\pi}{\lambda}$

$\Rightarrow \alpha = \frac{2\pi}{0.08} = 25\pi$

Also, $\beta = w = \frac{2\pi}{T}$

$\Rightarrow \beta = \frac{2\pi}{2} = \pi$

49. A famous relation in physics relates 'moving mass' m to 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A boy recalls the relationship almost correctly but forgets where to put the constant c. He writes: $m = \frac{m_0}{(1-v^2)^{1/2}}$

Guess where to put the missing c.

A) $m_0\left(1-\frac{v}{c}\right)^{v/2}$ **B)** $m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ **C)** $m_0\left(1-\frac{v}{c}\right)^{-1/2}$ **D)** $m_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$

SOL: From the given equation, $\frac{m_0}{m} = \sqrt{1-v^2}$

Since left hand side is dimensionless therefore right hand side should be also dimensionless.

So, $\sqrt{1-v^2}$ should be $\sqrt{1-\frac{v^2}{c^2}}$

The correct formula is $m = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$

Or

According to the principle of homogeneity of dimensions, powers of M, L, T on either side of the formula must be equal. For this, on RHS, the denominator $\left(1-v^2\right)^{1/2}$ should be dimensionless. Therefore, instead of $\left(1-v^2\right)^{1/2}$, we should write $\left(1-v^2/c^2\right)^{1/2}$. Hence, the correct formula would be

$m = \frac{m_0}{(1-v^2/c^2)^{1/2}}$

50. If $90° < A < 180°, 180° < B < 270°$ and $\cos A\frac{-\sqrt{3}}{2} = \sin B\frac{-3}{5} =$, then $\frac{2\tan B+\sqrt{3}\tan A}{\cot^2 A+\cos B}$ is

A) $\frac{3}{4}$ **B)** $\frac{4}{5}$ **C)** $-\frac{4}{5}$ **D)** $\frac{5}{22}$

SOL: A ← (90°, 180) 2nd quadrant

$\cos A = \frac{-\sqrt{3}}{2}\left[\frac{b}{h}\right]$

$\tan A = -\frac{1}{\sqrt{3}}$ $\cot A = \frac{1}{\tan A}$

B ← (180, 270°) 3rd quadrant

$\sin B = \frac{-3}{5}\left[\frac{p}{h}\right]$

$\tan B = \frac{3}{4}$

$\cos B = \frac{-4}{5}$

$$\frac{2\tan B+\sqrt{3}+\tan A}{\cot^2 A+\cos B}=\frac{2\times\frac{3}{4}+\sqrt{3}\times\frac{-1}{\sqrt{3}}}{3+(-415)}$$

$$=\frac{\frac{3}{2}-1}{3-\frac{4}{5}}$$

$$=\frac{3-2}{2\left[\frac{15-4}{5}\right]}=\frac{1}{2}\times\frac{5}{4}=\frac{5}{22}$$

51. The resistance $R=\frac{V}{i}$ where $V = 100 \pm 5$ volts and $i = 10 \pm 0.2$ *amperes*. then the total percentage of error in R_____.

SOL:

$R=\frac{V}{i}\Rightarrow V=100\pm5$

$i=10\pm0.2$

$R=\frac{V}{i}=\frac{100}{10}=10\,\Omega$

$\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta i}{i}$ $\quad \frac{\Delta V}{V}=\frac{5}{100}$ $\quad \frac{\Delta i}{i}=\frac{0.2}{10}$

$\frac{\Delta R}{R}\times100=5+2=7\%$

52. Temperature of source is 330°C. Temperature(in ^{0}k) of sink is changed in order to increase the efficiency of engine from $\frac{1}{5}$ to $\frac{1}{4}$, by_____

SOL:

$\eta=1-\frac{T_2}{T_1}$

$\frac{1}{5}=1-\frac{T_2}{T_1}\quad -①$

Similarly $\frac{1}{4}=1-\frac{T_2'}{T_1}\quad -②$

$②-①\Rightarrow\frac{1}{4}-\frac{1}{5}=1-\frac{T_2'}{T_1}-\left[1-\frac{T_2}{T_1}\right]$

$\frac{1}{4}-\frac{1}{5}=1-\frac{T_2'}{T_1}-1+\frac{T_2}{T_1}$

$\frac{5-4}{20}=\frac{T_2-T_2'}{T_1}$

$T_2-T_2'=\frac{1}{20}\times[330+273]$

$=30.15\ k$

53. A cylinder of fixed capacity 44.8 litre contains a monatomic gas at standard temperature and pressure. The amount of heat required to cylinder by 10°C will be_____R
(R = universal gas constant)

SOL:

We have $(\Delta Q)_V=\mu C_V\Delta T$, since the volume of cylinder is fixed.

As we know, 1 mol of any ideal gas at standard temperature and pressure occupies a volume of 22.4 litres

Hence, no. of moles of gas, $\mu=\frac{44.8}{22.4}$

Given $\Delta T=10^\circ C$

$\therefore (\Delta Q)_V=2\times\frac{3}{2}R\times10\quad[\because (C_V)_{mono}=\frac{3}{2}R]$

$=3R\times10$

$=30R.$

54. The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. The fundamental frequency of the sonometer wire in a Hz is

SOL: Frequency ∝ (Tension)1/2

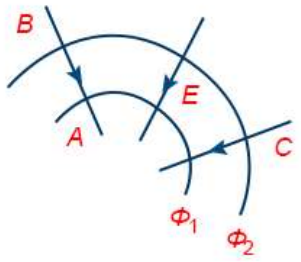
$$\frac{\Delta n}{n}\times100=\left[\left(\frac{121}{100}\right)^{1/2}-1\right]\times100$$

$$\frac{5}{n}\times100=\left[\frac{11}{10}-1\right]\times100$$

n = 50

Fundamental frequency of sonometer = 50 Hz

55. In moving form A to B along an electric field line, the electric field does $6.4\times10^{-19}\ J$ of work on an electron. If Φ_1, Φ_2 are equipotential surfaces, then the potential difference $(V_C - V_A)$ is(in V)___



SOL:

$V_B = V_C$

$V_B - V_A = V_C - V_A = \frac{W}{q_{V_0}} = \frac{6.4 \times 10^{-19}}{1.6 \times 10^{-19}}$

$V_B - V_A = +4V$

$V_B > V_A$ as field lines moves from higher potential to lower potential.

56. The energy level diagram for an hydrogen like atom is shown in the figure. The radius of its first Bohr orbit is_____Å.

SOL:

$r = \frac{n^2 h^2}{4\pi^2 m ke^2 . z}$

$r = \frac{0.53\ n^2}{z} \Rightarrow n = 1$

$\Delta E = 13.6\ z^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$

$\Rightarrow 54.4 = 13.6\ z^2\ (1)$

$\Rightarrow z^2 = 4 \Rightarrow z = 2$

$r = \frac{0.53(1)}{2} = 0.265$ A

57. The mass of a $_3Li^7$ nucleus is 0.042 a less than the sum of the masses of all its nucleons. The binding energy per nucleon of $_3Li^7$ nucleus is nearly _____MeV.

SOL:

$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + \Delta E$

The binding energy per nucleon of a deuteron = 1.1 Me V

$\therefore$ Total binding energy = 2 × 1.1 = 2.2 Me V

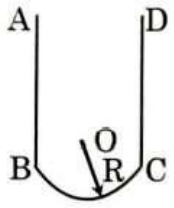
The binding energy per nucleon of a helium nuclei = 7 Me V

$\therefore$ Total binding energy = 4 × 7 = 28 Me V

Hence, energy released

$\Delta E = (28 - 2 \times 2.2) = 23.6$ Me V

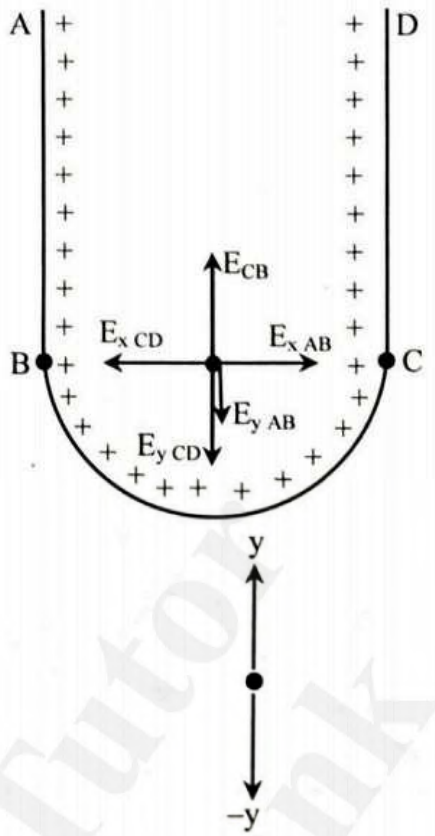
58.



A thread carrying a charge (uniform) λ per unit length has configuration shown in figure.

Assuming a curvature radius $\mathbf{R}$ to be considerably less than the length of thread. Find the magnitude of electric field strength at point $\mathbf{O}$

SOL:



Net $\vec{E}$ due to semicircular arc of radius R

$$E = \frac{2\lambda}{4\pi\varepsilon_0 R} = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} \text{ (+y direction)}$$

Net y component of $\vec{E}$ due to semi infinite line charges AB and CD in (–y) direction = E′

$$E' = \frac{\lambda}{4\pi\varepsilon_0 R} + \frac{\lambda}{4\pi\varepsilon R} = \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$$

X components of field of these line charges cancel each other.

$$E'' = E + E'$$

$$= \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j}) + \frac{\lambda}{2\pi\varepsilon_0 R}(+\hat{j})$$

$$= 0 \text{ (zero)}$$

59. An electromagnetic wave of frequency **5GHz**, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is **$\ldots\ldots \times 10^7$ m/s**.

SOL: Given, $\mu_r = \varepsilon_r = 2$

where, μ_r is relative permeability,

ε_r is relative permittivity.

Speed of electromagnetic wave v is given by

$v = \frac{c}{n}$

where, n = refractive index $= \sqrt{\mu_r \varepsilon_r} = \sqrt{4} = 2$

$\Rightarrow v = \frac{3\times10^8}{2} = 15 \times 10^7 \text{ m/s}$

$\because x \times 10^7 = 15 \times 10^7$

$\Rightarrow x = 15$

60. A compass needle oscillates 20 times per minute at a place where the dip is 30° and 30 times per minute where the dip is 60°. The ratio of total magnetic field due to the earth at two places respectively is $\frac{4}{\sqrt{x}}$. The value of x is

SOL: $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$T = 2\pi\sqrt{\frac{I}{MB\cos\theta}}$

$\frac{20}{30} = \sqrt{\frac{B_1}{B_2}\frac{\sqrt{3}/2}{1/2}}$

$\frac{4}{9} = \frac{B_1}{B_2}\sqrt{3}$

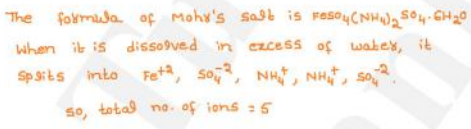
$\frac{B_1}{B_2} = \frac{4}{\sqrt{243}}$

Chemistry

61. How many ions per molecule are produced in the solution when mohr salt is dissolved in excess of water

A) 6 **B)** 4 **C)** 10 **D)** 5

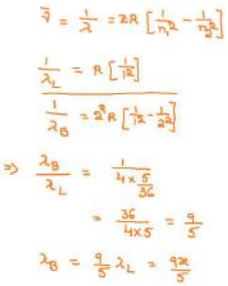
SOL:



62. If the shortest wavelength of hydrogen atom in Lyman series is x, then longest wavelength in Balmer series of He^+ is

A) $\frac{9x}{5}$ **B)** $\frac{36x}{5}$ **C)** $\frac{x}{4}$ **D)** $\frac{5x}{9}$

SOL:



63. Which of the following is not isostructural with $SiCl_4$?

A) PO_4^{3-} **B)** NH_4^+ **C)** SCl_4 **D)** SO_4^{2-}

SOL:

$SiCl_4$ — $H = \frac{4+4}{2} = \frac{8}{2} = 4$, sp^3 — tetrahedral

A) PO_4^{-3} — $H = \frac{5+3}{2} = \frac{8}{2} = 4$, sp^3 — tetrahedral

B) NH_4^+ — $H = \frac{5+4-1}{2} = \frac{8}{2} = 4$, sp^3 — tetrahedral

C) SCl_4 — $H = \frac{6+4}{2} = \frac{10}{2} = 5$, sp^3d — Distorted trigonal bipyramidal

D) SO_4^{-2} — $H = \frac{6+2}{2} = \frac{8}{2} = 4$, sp^3 — tetrahedral

64. If a gas at constant temperature and pressure expands, then its

A) internal energy decreases **B)** entropy increases and then decreases **C)** internal energy increases **D)** internal energy remains constant

SOL:

If a gas at constant temperature and pressure expands, then its internal energy remains constant.

$Q_p = \Delta H$

65. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

A) 9 **B)** 10 **C)** 11 **D)** 8

SOL: $K_{sp} = [Mg^{+2}][OH^-]^2$

$1\times10^{-11} = 10^{-3}\times[OH^-]^2$

$[OH^-]^2 = 10^{-8}$

$OH^- = 10^{-4}$

$p^{OH} = 4$ $[p^H + p^{OH} = 14]$

$p^H = 10$

66. The increasing order of atomic radii of the following group 13 elements is

A) Al < Ga < In < Tl **B)** Ga < Al < In < Tl **C)** Al < In < Ga < Tl **D)** Al < Ga < Tl < In

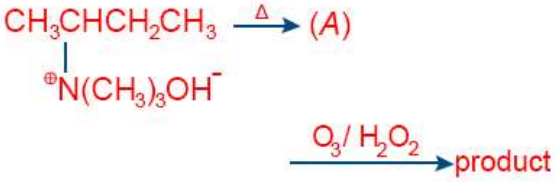
SOL: The correct order of atomic radii of group 13 elements is Al < In < Ga < Tl

67. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is

A) $Fe_4[Fe(CN)_6]_3$ **B)** $Na_3[Fe(CN)_6]$ **C)** $Fe(CN)_3$ **D)** $Na_4[Fe(CN)_5NOS]$

SOL: Ferric Ferrocyanide $Fe_4[Fe(CN_6)]_3$

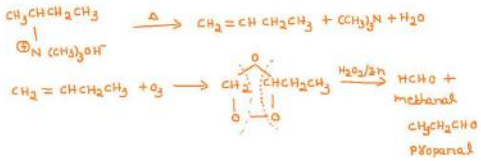
68. A major alkene (*A*) obtained in the following reaction undergo ozonolysis to give the product.



The product obtained is/are identified as

A) ethanal only **B)** methanal and propanal **C)** methanal and acetone **D)** Only acetone

SOL:



69. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be

A) 90.0 g mol^{-1} **B)** 115.0 g mol^{-1} **C)** 105.0 g mol^{-1} **D)** 210.0 g mol^{-1}

SOL:

We know that,
osmotic pressure of isotonic solutions are equal.
So, $\pi_1 = \pi_2$
$\Rightarrow c_1RT = c_2RT \quad [\because \pi = cRT]$
$\Rightarrow c_1 = c_2$
Given that the densities of both the solutions are equal, so both the substances are dissolved in the same volume of solvent.
$\therefore c_1 = \frac{5.25/M}{V}$
where M is the molecular weight of unknown substance and $c_2 = \frac{5.25/60}{V}$ [$\because$ Molar mass of urea = 60 g mol^{-1}]
$\therefore \frac{5.25/M}{V} = \frac{1.5/60}{V}$
$\Rightarrow M = \frac{60}{1.5} \times 5.25$
$\therefore M = 210$ g mol^{-1}

70. The hydrogen electrode is dipped in a solution of pH 3 at 25°C. The potential would be (the value of 2.303 RT/*F* is 0.059 V)

A) 0.177 V **B)** 0.087 V **C)** 0.059 V **D)** −0.177 V

SOL:

$E = E^0 + \frac{0.0591}{n}\log c$ $\quad pH = 3$
$= 0 + \frac{0.0591}{1}\log 10^{-3}$ $\quad [H^+] = 10^{-3}$
$= 0.0591 \times -3$
$= -0.1773$ V

71. For the reaction, $N_2O_5 \longrightarrow 2NO_2 + \frac{1}{2}O_2$

Give $\frac{-d[N_2O_5]}{dt} = K_1[N_2O_5]$; $\frac{d[NO_2]}{dt} = K_2[N_2O_5]$ and $\frac{d[O_2]}{dt} = K_3[N_2O_5]$, the relation between K_1 K_2 and K_3 is

A) $2K_1 = K_2 = 4K_3$ **B)** $K_1 = K_2 = K_3$ **C)** $2K_1 = 4K_2 = K_3$ **D)** None of these

SOL: $\frac{-d[NO_2]}{dt} = \frac{1}{2}\frac{d[NO_2]}{dt} = 2\frac{d[O_2]}{dt}$

$\therefore K_1[N_2O_5] = \frac{K_2}{2}[N_2O_5] = 2K_3[N_2O_5]$

$2K_1 = K_2 = 4K_3$

72. Which of the following is the most explosive?

A) NCl_3 **B)** PCl_3 **C)** $AsCl_3$ **D)** All

SOL:

$NCl_3 + 3H_2O \longrightarrow NH_3 + 3HOCl$

$HOCl \longrightarrow HCl + (O)$

So NCl_3 is most explosive

73. Misch metal is

A) an alloy of copper **B)** an alloy of lanthanoid metal **C)** an alloy of aluminium **D)** a mixture of chromium and lead chromate

SOL:

Misch metal is an alloy of Lanthanoid metal. It consists of typically 50% cerium, 25% lanthanum and 15-18 % neodymium.

74. Which of the following complex ions will not show optical activity?

A) $[Co(en)(NH_3)_2Cl_2]^+$ **B)** $[Cr(NH_3)_4Cl_2]^+$ **C)** $[Pt(Br)(Cl)(I)(NO_2)(Py)NH_3]$ **D)** cis-$[Co(en)_2Cl_2]^+$

SOL:

To show optical activity, at least one bidentate ligand should be there in the complex ion But for $[Cr(NH_3)_4Cl_2]^+$, there are no such ligands.

75. In the following groups:

1. —OAc 2. —OMe
3. $—OSO_2Me$ 4. $—OSO_2CF_3$

the order of leaving group ability is

A) 1 > 2 > 3 > 4 **B)** 4 > 3 > 1 > 2 **C)** 4 > 2 > 1 > 3 **D)** 2 > 3 > 4 > 1

SOL:

1) $-OAc$ 2) $-OMe$

3) $-OSO_2Me$ 4) $-OSO_2CF_3$

$4 > 3 > 1 > 2$

76. Which one of the following is more reactive than the rest towards a mixture of anhydrous. $ZnCl_2$ and concentrated HCl?

A) 2-methylpropan-2-ol **B)** methanol **C)** butan-2-ol **D)** butan-1-ol

SOL:

Anhydrous $ZnCl_2$ + Con HCl ⟶ Lucas reagent

2 - methylpropan - 2 - ol —— 3° alcohol
Methanol —— 1° alcohol
Butan - 2 - ol —— 2° alcohol
Butan - 1 - ol —— 1° alcohol

3° —— gives immediately turbidity with Lucas reagent
2° —— gives 5 to 10min turbidity with Lucas reagent
1° —— does not give turbidity with Lucas reagent

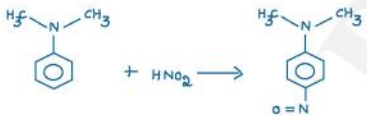
77. Identify the correct statement among the following.

A) n,n-dimethylaniline reacts with nitrous acid to give p-nitroso-N,N-dimethyl aniline **B)** bromination of p-toluidine produces 3,5-dibromo, 4-methlyaniline

C) aliphatic amines are less basic than ammonia **D)** aliphatic primary amines combine with nitrous acid under-cold conditions to form stable diazonium salts

SOL:

n,n – dimethylaniline, $C_8H_{11}N$, when gets reacted with nitrous acid give p-nitroso – N, N – dimethylaniline.



78. Which of the vitamins given below is water soluble?

A) Vitamin C **B)** Vitamin D **C)** Vitamin E **D)** Vitamin K

SOL: Vitamin C and B complex vitamins are water soluble.

79. Which of the following order is wrong?

A) $NH_3 < PH_3 < AsH_3$ —acidic **B)** $Li < Be < B < C$ —$(IE)_1$ **C)** $Al_2O_3 < MgO < Na_2O < K_2O$ —basic

D) $Li^{2+} < Na^+ < K^+ < Cs^+$ —ionic radius

SOL: The correct order of IE_1 in 2^{nd} period elements is $Li < Be > B < C$
Due to fully filled orbitals.

80. Equivalent weight of H_3PO_2 in a reaction is found to be half of its molecular weight. It can be due to its

A) reaction of its two H^+ ions **B)** oxidation to H_3PO_3 **C)** Both (a) and (b) **D)** None of the above

SOL:

(1) n factor is 2.
(2) Reaction with $2H^+$ ions.
(3) Oxidation of H_3PO_3.

81. 2s and 2p-atomic orbitals combine to give how many molecular orbitals ?

SOL: $2s \rightarrow (\sigma 2s)\ (\sigma^* 2s)$ (2)

$2p \rightarrow (\sigma 2p_z)\ (\sigma^* 2p_z)\ (\pi 2p_x)\ (\pi^* 2p_x)\ (\pi 2p_y)\ (\pi^* 2p_y)$ (6)

2S +2P = 2+6 = 8

82. If the enthalpy of vaporization of water is 186.5 kJ mol^{-1}, the entropy if its vaporization will be _____ JK^{-1} mol^{-1}

SOL: Given : $\Delta H = 186.5$ kJ mol^{-1}

B. pt of water = 100° C = 100 + 273 = 373 K

Entropy change, $\Delta S = \frac{\Delta H}{T} = \frac{186.5\ KJ\ mol^{-1}}{373K}$

$= 0.5$ kJ mol^{-1} K^{-1}

83. A weak acid, HA, has a K_a of 1.00×10^{-5}. If 0.0100 mole of this acid dissolved in one litre of water, the percentage of acid dissociated at equilibrium is closest to

SOL: Given $K_a = 1.00 \times 10^{-5}$, C = 0.100 mol for a weak electrolyte, degree of dissociation

$(\alpha) = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1\times10^{-5}}{0.100}} = 10^{-2} = 1\%$

84. If one third mole of permanganate oxidises 1.67 moles of M^{x+} as per the given reaction then the value of 'x' in the metal ion is _____

$MnO_4^- + M^{x+} \rightarrow Mn^{+2} + MO_3^- + \frac{1}{2}O_2$

SOL: Eq. of $MnO_7^- = Eq\ of\ M^{x+}$

$\frac{1}{3} \times 15 = 1.67 \times (5 - x)$

x = 2

85. The heat of hydrogenation of benzene is 50 kcal/mol. The resonance energy of benzene is 36 kcal/mol. The heat of hydrogenation of cyclohexene is approximately _____ kcal/mol

SOL:

Given resonance energy of benzene = 36 Kcal/mol
Then expected heat of hydrogenation of cyclohexatriene = 51+36
= 87 Kcal/mol
Now, heat of hydrogenation per double bond = $\frac{87}{3}$
= 29 Kcal/mol
∴ Required heat of hydrogenation of cyclohexene
= 29 Kcal/mol

86. The vapour pressure of a solution of 5 g of non electrolyte in 100 g of water at a particular temperature is 2985 Nm^{-2}. The vapour pressure of pure water at that temperature is 3000 Nm^{-2}. The molecular weight of the solute is _____.

SOL:

Given
wt of non-electrolyte, $W_2 = 5g$
wt. of water, $W_1 = 100g$
vapour pressure, $P_s = 2985\ N/m^2$
Vapour pressure of water, $P^0 = 3000\ N/m^2$
mol. wt of water, $M_1 = 18$
We know that

$$\frac{P^0 - P^s}{P^0} = \frac{W_2}{M_2} \times \frac{M_1}{W_1}$$

$$\Rightarrow \frac{3000 - 2985}{3000} = \frac{5}{M_2} \times \frac{18}{100}$$

$$\Rightarrow \frac{15}{3000} = \frac{5}{M_2} \times \frac{18}{100}$$

$$\Rightarrow \frac{1}{2} = \frac{5 \times 18}{M_2}$$

$$\Rightarrow M_2 = 180$$

$$\therefore M_2 = 180$$

87. On the basis of the information available from the reaction, $\frac{4}{3}Al + O_2 \rightarrow \frac{2}{3}Al_2O_3$,
$\Delta G = -827$ kJ mol^{-1} of O_2.
The minimum emf, required to carry out an electrolysis of Al_2O_3 is _____ V ($F = 96500$ C mol^{-1})

SOL:

Given
$\Delta G = -827\ kJ\ mol^{-1}$
$F = 96500\ C\ mol^{-1}$
We know that

$$\Delta G = -nEF \longrightarrow (1)$$

for 1 mol of Al, $n = 3$
given $\frac{4}{3}$ moles, so 1 mol — 3

$\frac{4}{3}$ mol — ?

$$= \frac{\frac{4}{3} \times 3}{1} = 4$$

So, $n = 4$
$\therefore$ from (1)

$$-827 \times 1000 = -4 \times E \times 96500$$

$$\Rightarrow E = \frac{827 \times 1000}{4 \times 96500}$$

$$= 2.142$$

$$\therefore E = 2.14\ V$$

88. In a certain gaseous reaction $A \longrightarrow B$, the initial pressure is 214 atm and the rate constant is $2.303 \times 10^{-4}\ s^{-1}$. What would be pressure (in atm) of A after 5 mins?
[Given: $10^{0.03} = 1.07$]

SOL: Initial pressure $P_i = 214$ atm Rate constant
$k = 2.303 \times 10^{-4}\ s^{-1}$
Unit of rate constant indicates first order reaction.
Time, $t = 5$ minutes $= 5 \times 60$ s
The integrated rate law is,

$$k = \frac{2.303}{t} \log_{10}\left(\frac{P_i}{P_f}\right)$$

$$2.303 \times 10^{-4} = \frac{2.303}{5 \times 60} \log_{10}\left(\frac{214}{P_f}\right)$$

$$\therefore \quad 0.03 = \log_{10}\left(\frac{214}{P_f}\right)$$

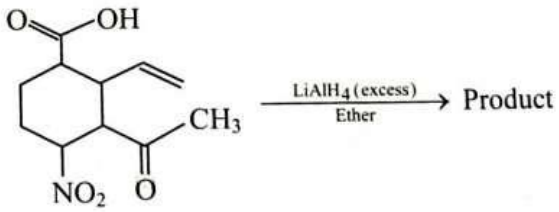
$$\therefore \quad 1.07 = \frac{214}{P_f}$$

$$\therefore \quad P_f = \frac{214}{1.07} = 200\ \text{atm}$$

89. How many of the following oxides are amphoteric in nature? $N_2O_3, P_2O_3, As_2O_3, Sb_2O_3, Bi_2O_3$

SOL: Acidic nature of each type decreases from **N** to **Bi**.
As_2O_3 and Sb_2O_3 are amphoteric

90.

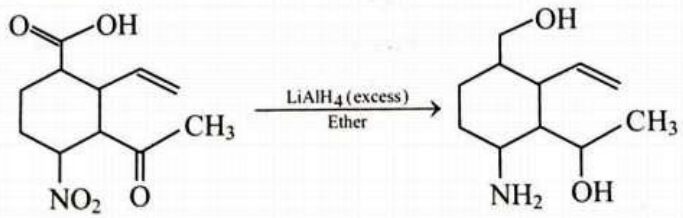


The number of π bonds in the major product will be _____.

SOL:

$LiAlH_4$ reduces aldehydes, ketones and carboxylic acids to corresponding alcohols and nitro ($-NO_2$) group to amino ($-NH_2$) group.

But $LiAlH_4$ cannot reduce carbon-carbon double bonds.



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