Date : 06/03/2024
Time : 3 Hours 0 Minutes

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1. $\left(1+cos\frac{\pi}{8}\right)\left(1+cos\frac{3\pi}{8}\right)\left(1+cos\frac{5\pi}{8}\right)\left(1+cos\frac{7\pi}{8}\right)=$

A) 1/2 **B)** 1/4 **C)** 1/8 **D)** 1/16

SOL:

Given

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left[1-\cos\left(\pi-\frac{5\pi}{8}\right)\right]\left[1-\cos\left(\pi-\frac{7\pi}{8}\right)\right]$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left[1-\cos\left(\frac{8\pi-5\pi}{8}\right)\right]\left[1-\cos\left(\frac{8\pi-7\pi}{8}\right)\right]$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$=\sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8}$

$=\sin^2\left(22\frac{1}{2}^\circ\right)\cdot\sin^2\left(67\frac{1}{2}^\circ\right)$

$=\frac{\sqrt{2}-1}{2\sqrt{2}}\times\frac{\sqrt{2}+1}{2\sqrt{2}}$

$=\frac{(\sqrt{2})^2-1}{(2\sqrt{2})^2}=\frac{1}{8}$

2. If 1+ sin x + $\sin^2$x + ... ∞ = $4+2\sqrt{3}$, 0 < x < π and $x\neq\frac{\pi}{2}$, then x =

A) $\frac{\pi}{3},\frac{2\pi}{3}$ **B)** $\frac{\pi}{6},\frac{\pi}{3}$ **C)** $\frac{\pi}{3},\frac{5\pi}{6}$ **D)** $\frac{2\pi}{3},\frac{\pi}{6}$

SOL:

Given $1+\sin x+\sin^2 x+\dots+\infty=4+2\sqrt{3}$, $0<x<\pi$ and $x\neq\frac{\pi}{2}$

$1+\sin x+\sin^2 x+\dots+\infty$

$\Rightarrow \frac{1}{1-\sin x}=4+2\sqrt{3}$

$\Rightarrow 1-\sin x=\frac{1}{2(2+\sqrt{3})}$

$\Rightarrow \sin x=\frac{4+2\sqrt{3}-1}{2(2+\sqrt{3})}$

$\Rightarrow \sin x=\frac{\sqrt{3}}{2}$

$\Rightarrow x=\frac{\pi}{3},\frac{2\pi}{3}$

3. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the A.M. of a and b, then $n=$

A) 1 **B)** −1 **C)** 0 **D)** None of these

SOL:

Given $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the arithematic mean of a & b is $\therefore \frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

$\Rightarrow 2(a^{n+1}+b^{n+1})=a\cdot a^n+b\cdot b^n+ba^n+ab^n$

$\Rightarrow 2a^{n+1}+2b^{n+1}=a^{n+1}+b^{n+1}=ba^n+ab^n$

$\Rightarrow a^{n+1}-ab^n+b^{n+1}-ba^n=0$

$\Rightarrow (a-b)(a^n-b^n)=0$

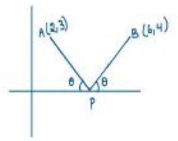
if $a^n-b^n=0$ then $\left(\frac{a}{b}\right)^n=1=\left(\frac{a}{b}\right)^0$

$\therefore n=0$

4. A ray of light is sent along the line which passes through the point (2, 3). The ray is reflected from the point P on x-axis. If the reflected ray passes through the point (6, 4), then the

A) $\left(\frac{26}{7},0\right)$ **B)** $\left(0,\frac{26}{7}\right)$ **C)** $\left(-\frac{26}{7},0\right)$ **D)** none of these

SOL:



Slope of AP $= \frac{-3}{x-2}$

$\therefore \tan\theta = \frac{-3}{x-2}$,

PB slope $\Rightarrow \tan\theta = \frac{-4}{x-6}$

$\frac{-3}{x-2} = \frac{-4}{x-6}$

$18 - 3x = 4x - 8$

$7x = 26$

$x = \frac{26}{7}$

$\therefore P\left(\frac{26}{7}, 0\right)$

5. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line drawn from the point P intersects the curve at points Q and R. If the product PQ. PR is independent of t

A) parabola **B)** circle **C)** ellipse **D)** none of these

SOL:

let $P(x_1, y_1)$ be a point not lying on $ax^2 + 2hxy + by^2 = 1$ and θ is inclination through P which intersects the given curve at Q and R.

The equation of line through P is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

$$a(x_1 + r\cos\theta)^2 + 2h(x_1 + r\cos\theta)(y_1 + r\sin\theta) + b(y_1 + r\sin\theta)^2 = 1$$

$$(a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta)r^2 + 2(ax_1\cos\theta + hx_1\sin\theta + hy_1\cos\theta + by_1\sin\theta)r + (ax_1^2 + 2hx_1y_1 + by_1^2 - 1) = 0$$

$$PQ\,PR = \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta}$$

$$= \frac{ax_1^2 + 2hx_1y_1 + by_1^2 - 1}{\left(\frac{a+b}{2}\right) + h\sin 2\theta + \left(\frac{a-b}{2}\right)\cos 2\theta}$$

which is independent of θ if $h = 0$ and $\frac{a-b}{2} = 0$

$\Rightarrow h = 0,\ a = b$

$\Rightarrow$ circle

6. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperb

A) $1 - \sqrt{2/3}$ **B)** $\sqrt{3/2} - 1$ **C)** $1 + \sqrt{2/3}$ **D)** $\sqrt{3/2} + 1$

SOL:

Given Hyperbola Equation.

$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$

$x^2 - 2\sqrt{2}x - 2(y^2 + 2\sqrt{2}y) = 6$

$x^2 - 2\sqrt{2}x + (\sqrt{2})^2 - 2 - 2(y^2 + 2\sqrt{2}y + (\sqrt{2})^2) + 4 = 6$

$(x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$

$\frac{(x-\sqrt{2})^2}{4} - \frac{2(y+\sqrt{2})^2}{4} = 1$

$\frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$

$\therefore$ eccentricity $(e) = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{4+2}{4}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}}$

Area formed by vertex, length of latus rectum, focus is $e - 1 = \sqrt{\frac{3}{2}} - 1$

7. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

A) a function **B)** transitive **C)** not symmetric **D)** reflexive

SOL:

$R = \{(1,3), (4,2), (2,4), (2,3), (3,1)\}$

R to be reflexive $(a, a) \in R$

$(1,1) \notin R$ so R is not reflexive

To be Symmetric $(a,b) \in R \Rightarrow (b,a) \in R$

$(1,3)$ and $(3,1) \in R$ | but $(3,2) \notin R$ as

$(2,4)$ and $(4,2) \in R$ | $(2,3) \in R$

So 'R' is not Symmetric

8. The product of matrices $A = [cos2\theta cos\theta sin\theta cos\theta sin\theta sin2\theta] \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $\sin B = [cos2\phi cos\phi sin\phi cos\phi sin\phi sin2\phi] \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is a null n

A) $2n\pi, n \in Z$ **B)** $n\frac{\pi}{2}$, n ∈ Z **C)** $(2n+1)\frac{\pi}{2}$, n ∈ Z **D)** nπ, n ∈ Z

SOL:

Given $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

$A\cdot B = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\sin\theta\cdot\cos\phi\sin\phi & \cos^2\theta\cdot\cos\phi\sin\phi + \cos\theta\sin\theta\cdot\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$

$= \begin{bmatrix} \cos\theta\cdot\cos\phi[\cos(\theta-\phi)] & \cos\theta\sin\phi[\cos(\theta-\phi)] \\ \sin\theta\cdot\cos\phi[\cos(\theta-\phi)] & \sin\theta\sin\phi[\cos(\theta-\phi)] \end{bmatrix}$

$= \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\theta\sin\phi \end{bmatrix}$

Now $A\cdot B = 0 \Rightarrow \cos(\theta-\phi) = 0$

$\Rightarrow \theta-\phi = (2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$

9. If $y^3 - y = 2x$, then $\left(x^2 - \frac{1}{27}\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$

A) y **B)** $\frac{y}{3}$ **C)** $\frac{y}{9}$ **D)** $\frac{y}{27}$

SOL:

$y^3 - y = 2x$

$(3y^2-1)\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{3y^2-1}$

$\frac{d^2y}{dx^2} = \frac{-2}{(3y^2-1)^2}\cdot(6y)\frac{dy}{dx}$

$= \frac{-12y}{(3y^2-1)^2}\cdot\frac{2}{(3y^2-1)}$

$= \frac{-24y}{(3y^2-1)^3}$

$\therefore \left(x^2-\frac{1}{27}\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$\left(x^2-\frac{1}{27}\right)\left[\frac{-24y}{(3y^2-1)^3}\right] + x\left(\frac{2}{3y^2-1}\right)$

By replacing $x = \frac{y^3-y}{2}$, we get the value $\frac{y}{9}$

10. An artillery target may be either at point I with the probability $\frac{8}{9}$ or at the point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each she

A) 9 **B)** 10 **C)** 11 **D)** 12

SOL:

A = event that target is hit when x shells are fired at point I

Let E_1 & E_2 denote the events hitting I & II respectively.

$P(E_1) = \frac{8}{9}$, $P(E_2) = \frac{1}{9}$

$P(A/E_1) = 1 - \left(\frac{1}{2}\right)^x$ $\quad P(A/E_2) = 1 - \left(\frac{1}{2}\right)^{21-x}$

Now $P(A) = \frac{8}{9}\left[1-\left(\frac{1}{2}\right)^x\right] + \frac{1}{9}\left[1-\left(\frac{1}{2}\right)^{21-x}\right]$

$\therefore \frac{dP(A)}{dx} = \frac{8}{9}\left[\left(\frac{1}{2}\right)^x\cdot\log 2\right] + \frac{1}{9}\left[-\left(\frac{1}{2}\right)^{21-x}\log 2\right]$

For Max. probability $\frac{\partial P(A)}{\partial x} = 0 \Rightarrow 2^{3-x} = 2^{x-21}$

$\Rightarrow x = 12$

Since $\frac{d^2P(A)}{dx^2} < 0$ for $x = 12$

P(A) is max when $x = 12$

11. If the integral $\int\frac{5\tan x}{\tan x - 2}dx = x + a\,ln\,|\sin x - 2\cos x| + k$, then a is equal to:

A) -1 **B)** -2 **C)** 1 **D)** 2

SOL:

$\int\frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + K$

put $5\sin x = A(\sin x - 2\cos x) + B\cdot\frac{d}{dx}(\sin x - 2\cos x)$

$= \sin x(A+2B) + \cos x(-2A+B)$

$A + 2B = 5$ and $B = 2A$

$A = 1$ and $B = 2$

$\int\frac{5\tan x}{\tan x - 2}dx = x + 2\ln|\sin x - 2\cos x|$

$a = 2$

12. The area of the portion of the circle $x^2 + y^2 = 1$, which lies inside the parabola $y^2 = 1 - x$, is

A) $\frac{\pi}{2} - \frac{2}{3}$ **B)** $\frac{\pi}{2} + \frac{2}{3}$ **C)** $\frac{\pi}{2} + \frac{4}{3}$ **D)** $\frac{\pi}{2} - \frac{4}{3}$

SOL:

$x^2+y^2=1$, $y^2=1-x$

$x^2+1-x=1$

$\Rightarrow x^2-x=0$

$\Rightarrow x(x-1)=0$

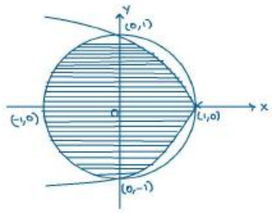
$\Rightarrow x=0$ (or) 1

$\text{Area}(A)=\frac{\pi}{2}+2\int_0^1\sqrt{1-x}\,dx$

$=\frac{\pi}{2}+2\left(\frac{-2}{3}(1-x)^{3/2}\right)_0^1$

$=\frac{\pi}{2}+\frac{2}{3}(2)$ Sq units.

$=\frac{\pi}{2}+\frac{4}{3}$.



13. If the pth, qth and rth terms of a G.P. are positive numbers a, b and c, respectively, then the angle between the vectors $il_na + jl_nb + kl_nc$ and $i(q-r) + j(r-p) + k(p-q)$ is

A) $\frac{\pi}{3}$ **B)** $\frac{\pi}{6}$ **C)** $\frac{\pi}{2}$ **D)** none of these

SOL:

Let $p=1$, $q=2$, $r=3$

$a=1$, $b=2$, $c=4$

$\cos\theta=\left(\bar{i}(\ln 1)+\bar{j}(\ln 2)+\bar{k}(\ln 4)\right)$

$=\dfrac{\left(i(-1)+\bar{j}(2)+\bar{k}(-1)\right)}{\sqrt{(\ln 1)^2+(\ln 2)^2+(\ln 4)^2}\sqrt{(1)^2+2^2+1^2}}$

$=\dfrac{2(\ln 2)-\ln 4}{\sqrt{(\ln 2)^2+(\ln 4)^2}\sqrt{6}}$

$=0$

14. The shortest distance between the lines given by $r=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$ and $r=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$ is

A) 84 **B)** 14 **C)** 21 **D)** 16

SOL:

$r=(8+3\lambda)\hat{i}-(9+16\lambda)\hat{j}+(10+7\lambda)\hat{k}$

$=(8\hat{i}-9\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$

$r=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

$(3,-16,7)\times(3,+8,-5)$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & +8 & -5\end{vmatrix}=\hat{i}(80-56)-\hat{j}(-15-21)+\hat{k}(24+48)$

$=24\hat{i}+36\hat{j}+72\hat{k}$

$=12(2\hat{i}+3\hat{j}+6\hat{k})$

$|24\hat{i}+36\hat{j}+72\hat{k}|=12(7)=84$

$(24,36,72)\cdot(7,38,-5)$

$=12[(2,3,6)\cdot(7,38,-5)]=12(14+114-30)$

$=12(98)$

$\therefore$ Distance $=\frac{12\times 98}{84}=14$

15. A letter is know to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that is come from LONDON is

A) $\frac{5}{17}$ **B)** $\frac{12}{17}$ **C)** $\frac{17}{30}$ **D)** $\frac{3}{5}$

SOL:

A_1 = Selecting from LONDON

A_2 = Selecting from CLIFTON

E = Selecting ON

$P(A_1\cap E)=\frac{2}{5}$, $P(A_2\cap E)=\frac{1}{6}$

Required $=P\left(\frac{A_1}{E}\right)$

$=\dfrac{P(A_1\cap E)}{P(A_1\cap E)+P(A_2\cap E)}$

$=\dfrac{\frac{2}{5}}{\frac{2}{5}+\frac{1}{6}}=\frac{12}{17}$.

16. Let Z denote the set of all integers where $A=\left\{(a,b):a^2+3b^2=28, a,b\in Z\right\}$ and $B=\{(a,b):a>b, a,b\in Z\}$, then the number of elements in $A\cap B$ is

A) 2 **B)** 6 **C)** 4 **D)** 5

SOL: $A=\left\{(a,b):a^2+3b^2=28, a,b\in Z\right\}$

(a, b) can be (1, 3), (−1, 3), (1, −3), (−1, −3), (5, 1)
(−5, 1), (5, −1), (−5, −1), (4, 2), (−4, 2), (4, −2), (−4, −2)

$n(A) = 12, n(B) = \infty$

$A \cap B = \{(a, b) : a^2 + 3b^2 = 28 \text{ and } a > b, a, b \in Z\} = 6$

17. If the mean deviation of the numbers 1 . $1 + d, \ldots, 1 + 100d$ from their mean is $255,$ then d is equal to

A) 10.1 **B)** 20.2 **C)** 10.0 **D)** 20.0

SOL: $\bar{x} = \frac{1}{101}[1 + (1 + d) + (1 + 2d) + \ldots\ldots + (1 + 100d)]$
$= \frac{1}{101} \times \frac{101}{2}[1 + (1 + 100d)] = 1 + 50d$
$\therefore$ Mean deviation from mean
$= \frac{1}{101}[11 - (1 + 50d)\ | + 11 + d - (1 + 50d)| + \ldots\ldots + |1 + 100d - (1 + 50d)\ |]$
$= \frac{2|d|}{101}\{1 + 2 + \ldots\ldots + 50]$
$\frac{2|d|}{101} \frac{50(51)}{2} = \frac{2550}{101}|d|$
Now, $\frac{2550}{101}|d| = 255 \Rightarrow |d| = 10.1$
Thus, we may take $d = 10.1$

18. Statement I Polar form of $\frac{1+7i}{(2-i)^2}$ is $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$ Statement II Polar form of $\frac{1+3i}{1-2i}$ is $\sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

A) Statement I is correct **B)** Statement II is correct **C)** Both are correct **D)** Neither I nor II is correct

SOL: First we will convert the given expressions into $a + ib$ form and then reduce them into polar form

$1.Letz = 1 + 7i(2 - i)2 = 1 + 7i4 + i2 - 4i = 1 + 7i4 - 1 - 4i[use(a - b)2 = a2 + b2 - 2ab] = 1 + 7i3 - 4i \times 3 + 4i3 + 4i = 3 + 4i + 21i + 28i2(3)2 - (4i)2[\because (c$

Now, let $-1 + i = r\cos\theta + ir\sin\theta$
On comparing the real and imaginary parts of both sides, we get
$r\cos\theta = -1$
and $r\sin\theta = 1$
Squaring and adding Eqs. (i) and (ii), we get

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (1)^2$

$r2cos2\theta + r2sin2\theta = (-1)2 + (1)2r2(cos2\theta + sin2\theta) = 1 + 1 \Rightarrow r2 = 2(\because cos2\theta + sin2\theta = 1)$ $r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$

$\Rightarrow \quad r^2 = 2 \quad \left(\because \cos^2\theta + \sin^2\theta = 1\right)$

$\Rightarrow$
$r = \sqrt{2}$
On dividing Eq. (ii) by Eq. (i), we get
$\frac{r\sin\theta}{r\cos\theta} = \left|\frac{1}{-1}\right| \Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4}$
$\Rightarrow \quad \theta = \frac{\pi}{4}$
Since, the real part of z is negative and the imaginary part of z is positive. So, the point lies in II quadrant.
$\therefore \quad \arg(z) = \pi - \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$
$\therefore$
$z = -1 + i = \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$
which is the required polar form of $\frac{1+7i}{(2-i)^2}$

L

$Letz = 1 + 3i1 - 2i \times 1 + 2i1 + 2i = 1 + 2i + 3i + 3i \times 2i12 - (2i)2 = 1 + 5i + 6i21 - 4i2[\because (a + b)(a - b) = a2 - b2] = 1 + 5i - 61 + 4 = -5 + 5i5 = -1 + i$

19. A unit vector a makes an angle $\frac{\pi}{4}$ with Z -axis, if $\hat{\mathbf{a}} + \hat{\mathbf{i}} + \hat{\mathbf{j}}$ is a unit vector, then a is equal to

A) $\frac{\hat{\mathbf{i}}}{2} + \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{2}$ **B)** $\frac{\hat{\mathbf{i}}}{2} + \frac{\hat{\mathbf{j}}}{2} - \frac{\hat{\mathbf{k}}}{\sqrt{2}}$ **C)** $-\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$ **D)** $\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} - \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

SOL: Let $\mathbf{a} = l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + n\hat{\mathbf{k}}$ makes an angle $\frac{\pi}{4}$ with Z-axis

Also, $l^2 + m^2 + n^2 = 1$

Here, $n = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, l^2 + m^2 = \frac{1}{2}$ ...(i)

$\therefore \quad \mathbf{a} = l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Rightarrow \therefore \quad \mathbf{a} + \hat{\mathbf{i}} + \hat{\mathbf{j}} = (l+1)\hat{\mathbf{i}} + (m+1)\hat{\mathbf{j}} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Rightarrow \quad |\mathbf{a} + \hat{\mathbf{i}} + \hat{\mathbf{j}}| = \sqrt{(l+1)^2 + (m+1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow \quad 1 = l^2 + m^2 + 2 + 2l + 2m + \frac{1}{2}$

$\Rightarrow \quad l + m = -1$ [from Eq. (i)]

$\Rightarrow \quad l^2 + m^2 + 2lm = 1$

$\Rightarrow \quad 2lm = \frac{1}{2} \Rightarrow l = m = -\frac{1}{2}$

$\left(\because l = m = \frac{1}{2} \text{ is not satisfied the given equation}\right)$

$\therefore \quad \mathbf{a} = -\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

20. Let $f : R \to R$ be a continuous function. Then, $\lim_{x \to \frac{\pi}{4}} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to

A) $f(2)$ **B)** $2f(2)$ **C)** $2f(\sqrt{2})$ **D)** $4f(2)$

SOL: Using L-Hopital's rule

$$\lim_{x \to \frac{\pi}{4}} \frac{\frac{\pi}{4} \cdot 2\sec x \cdot \sec x \cdot \tan x \cdot f(\sec^2 x) - 0}{2x}$$

[using Leibnitz theorem]

$$= \frac{\frac{\pi}{4} \cdot 2(\sqrt{2})^2 \cdot (1) f(2)}{2 \cdot \frac{\pi}{4}} = 2f(2)$$

21. The value of tan 9° − tan 27° − tan 63° + tan 81° is

SOL:

Value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = 2\left[\frac{\sin 54° - \sin 18°}{\sin 54° \sin 18°}\right]$

$= 2\left[\frac{2\cos 36° \sin 18°}{\sin 18° \cos 36°}\right]$

$= 4$

22. If $x^2 - x + 1 = 0$, then find value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$.

SOL: $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{3}i}{2} = -\omega, -\omega^2$

$\therefore \sum_{n=1}^{5}\left(x^{2n} + \frac{1}{x^{2n}} + 2\right)$

$\Rightarrow \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 \frac{1}{x^4} + 2\right) + \left(x^6 \frac{1}{x^6} + 2\right) + \left(x^8 \frac{1}{x^8} + 2\right) + \left(x^{10} \frac{1}{x^{10}} + 2\right)$

$\Rightarrow (\omega^2 + \omega^4 + \omega^6 + \omega^8 + \omega^{10}) + \left(\frac{1}{\omega^2} + \frac{1}{\omega^4} + \frac{1}{\omega^6} + \frac{1}{\omega^8} + \frac{1}{\omega^{10}}\right) + 10$

$\Rightarrow -1 -1 + 10 = 8.$

23. The number of 3 digit numbers having atleast one of their digit as 5 are

SOL: Total number of 3-digit numbers having atleast one of their digits as 5 = Total number of 3-digit numbers - (Total number of 3-digit numbers in which 5 does not appear at all)

= 9 × 10 × 10 − 8 × 9 × 9
= 900 − 648 = 252

24. If the number of terms in $\left(x+1+\frac{1}{x}\right)^n\,(n \in I^+)$ is 401 then value of n is

SOL:

Given $\left(x+1+\frac{1}{x}\right)^n$ $(n \in I^+)$ is 401.

$\left(x+1+\frac{1}{x}\right)^n = \frac{(1+x+x^2)^n}{x^n}$

$\therefore (x^2+x+1)^n$ is the form

$a_0+a_1x+a_2x^2+\cdots+a_{2n}x^{2n}$

which contains $2n+1$ terms

$\therefore 2n+1=401$

$2n=400$

$n=200$.

25. If PSQ is the focal chord of the parabola $y^2 = 8x$ such that SP = 6. Then the length of SQ is

SOL: Since the semi latus rectum of a parabola is the harmonic mean between the segments of any focal chord of a parabola, therefore.

SP, 4, SQ are in HP.

$\Rightarrow 4 = \frac{2\,SP.SQ}{SP+SQ}$

$\Rightarrow 4 = \frac{2\times 6\times SQ}{6+SQ}$

24 + 4.SQ = 12.SQ

8 SQ = 24

SQ = 3.

26. The real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4y\frac{dy}{dx}+y^4=4x^6$ into a homogeneous equation is

SOL:

Given Equation $2x^4y\frac{dy}{dx}+y^4=4x^6$

$2x^4.u^m.m.u^{m-1}\frac{du}{dx}+u^{4m}=4x^6$

$2m.x^4.u^{2m-1}\frac{du}{dx}+u^{4m}=4x^6$

In any term degree must be '6' to have homogeneous equation

$\therefore 4m=6$

$m=\frac{3}{2}$

To convert equation to homogemeneous

$y=u^m$ "m" must be $\frac{3}{2}$.

27. The probability that in a year of the 22nd century chosen at random there will be 53 Sundays is

SOL: We know a leap year is fallen within 4 years, so its probability = $\frac{1}{4}$

53rd Sunday in a leap year = $\frac{1}{4}\times\frac{2}{7}=\frac{2}{28}$

Similarly probability of 53rd Sunday in a non-leap year = $\frac{3}{4}\times\frac{1}{7}=\frac{3}{28}$

∴ required probability = $\frac{2}{28}+\frac{3}{28}=\frac{5}{28}=0.1785$

28. If $\lim_{x\to 0}\frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}=\frac{11e}{k}$ then find k

SOL: Using expansion

$\lim_{x\to 0}\frac{\left(e-\frac{ex}{2}+\frac{11e}{24}x^2\ldots\ldots\right)-e+\frac{ex}{2}}{x^2}=\frac{11e}{24}$

29. If $0\le[x]<2, -1\le[y]<1$ and $1\le[z]<3$ where [.] denotes greatest integral function then the maximum value of the determinant. $D=\Big|[x]+1[y][z][x][y]+1[z][x][y][z]+$

SOL:

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix}$$

solving = $[x] + [y] + [z] + 1$

taking maximum value we get 4, note that $[x]$ is always an integer

30. If $\int \frac{\cos 4x+1}{\cot x-\tan x} dx = -\frac{1}{\lambda}\cos 4x + B$, then find value of λ.

SOL: $\int \frac{2\cos^2 2x-1+1}{\frac{\cos 2x}{\sin x \cdot \cos x}} dx = \int \sin 2x \cdot \cos 2x dx$

$= \frac{1}{2}\int \sin 4x dx$

$= -\frac{1}{8}\operatorname{cosec} x + C$

31. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (i) the total mass of the box (ii) the differe

A) 2.8 kg, 0.08 g **B)** 2.9 kg, 0.02 g **C)** 2.3 kg, 0.02 g **D)** 3.0 kg, 0.02 g

SOL: Given, mass of the box $m = 2.3$ kg
Mass of first gold piece, $m_1 = 20.15$ g $= 0.02015$ kg
Mass of second gold piece $m_2 = 20.17$ g $= 0.02017$ kg
Total mass of the box, $M = m + m_1 + m_2 = 2.3 + 0.02015 + 0.02017$
$= 2.34032$ kg
As the mass of the box has least decimal place, i.e., one decimal place, therefore total mass of the box can have only one decimal place.
Rounding off the total mass of the box upto one decimal place, we get
(i) Total mass of the box, $M = 2.3$ kg
(ii) Difference in masses of gold pieces,
$\Delta m = m_2 - m_1 = 20.17 - 20.15 = 0.02$ g
The masses of two gold pieces has two decimal places, therefore it is correct upto two places of decimal.

32. A Carnot's reversible engine converts 1/6 heat input into work. When the temperature of sink is reduced by 62 K, the efficiency of Carnot's cycle becomes 1/3. The temperatures of

A) 350 K, 300 K **B)** 390 K, 330 K **C)** 372 K, 310 K **D)** None of these

SOL:

We have $\eta = \frac{T_1 - T_2}{T_1}$

Given $\eta = 1/6$, $\eta_2 = 1/3$, In Second Case Temp is reduced to 62 K

In first case,

$\therefore \frac{1}{6} = \frac{T_1 - T_2}{T_1}$

$T_1 = 6T_1 - 6T_2$

$5T_1 - 6T_2 = 0$ —(i)

In Second case, $\frac{1}{3} = \frac{T_1 - (T_2 - 62)}{T_1}$

$T_1 = 3T_1 - 3T_2 + 186$

$2T_1 - 3T_2 = -186$ —(ii)

multiply (i) with 2 and (ii) with 5,

by solving (i) and (ii) we get

$10T_1 - 12T_2 = 0$

$10T_1 - 15T_2 = -930$

$3T_2 = 930$

$T_2 = 310$ K

Then $5T_1 = 1860$

$T_1 = 372$ K

33. An insulated container of gas has two chambers separated by an insulating partition. one of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. Tl

A) $\frac{T_1T_2(p_1V_1+p_2V_2)}{p_1V_1T_2+p_2V_2T_1}$ **B)** $\frac{p_1V_1T_1+p_2V_2T_2}{p_1V_1+p_2V_2}$ **C)** $\frac{p_1V_1T_2+p_2V_2T_1}{p_1V_1+p_2V_2}$ **D)** $\frac{T_1T_2(p_1V_1+p_2V_2)}{p_1V_1T_1+p_2V_2T_2}$

SOL:

If two vessels Containing air at T_1 & T_2 K are pressure P_1 & P_2 are mixed

$n_1 + n_2 = n_1' + n_2'$

$\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2} = \frac{P_1V_1 + P_2V_2}{T}$

$\frac{P_1V_1T_2 + P_2V_2T_1}{T_1T_2} = \frac{P_1V_1 + P_2V_2}{T}$

$T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$

34. A simple pendulum has a time period of 3.0 s. If the point of suspension of the pendulum starts moving vertically upward with a velocity v = kt where k = 4.4 ms^{-2}. The new time p

A) $\frac{9}{4}$S **B)** $\frac{5}{3}$S **C)** 2.5 S **D)** 4.4 S

SOL: Original time period is

$T_1 = 2\pi\sqrt{\frac{l}{g}}\ .\ .\ .\ .\ .\ (i)$

When the pendulum is moving upwards, the effective value of g is $g_{eff} = g+a$

Where a is the acceleration of the pendulum which is given by

$a = \frac{dv}{dt} = \frac{d}{dt}(kt)$

$k = 4.4ms^{-1}$

Therefore, the new time period is

$T_2 = 2\pi\sqrt{\frac{l}{g_{eff}}}\ .\ .\ .\ .\ (ii)$

From (1) and (2) we get

$\frac{T_1}{T_2} = \sqrt{\frac{g}{g_{eff}}} = \sqrt{\frac{10}{14.4}} = \frac{1}{1.2}$

$T_2 = \frac{T_1}{1.2} = \frac{3}{1.2} = 2.5S$

35. The torque acting on a dipole of moment $\vec{P}$ in an electric field $\vec{E}$ is

A) $\vec{P}\cdot\vec{E}$ **B)** $\vec{P}\times\vec{E}$ **C)** Zero **D)** $\vec{E}\times\vec{P}$

SOL:

$\vec{P} \times \vec{E}$

Expression for torque in the vector form is

where P is dipole moment

E is electric field

$\tau = \vec{P} \times \vec{E}$

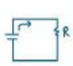
36. Let C be the capacitance of a capacitor discharging through a resistor R. suppose t_1 is the time taken for the energy stored in the capacitor to reduced to half its initial value and t_2 is

A) 1 **B)** 1/2 **C)** 1/4 **D)** 2

SOL: Energy stored in capacitor $E = \frac{1}{2}\frac{q^2}{c}$

$\Rightarrow q = q_0 e^{-t/Rc}$ —(1)

$\Rightarrow E = \frac{1}{2}\frac{[q_0 e^{-t/Rc}]^2}{c}$



$E = \frac{q_0^2 e^{-2t/Rc}}{2c}$

at $t = 0$ [initial condition]

$E = E_i = \frac{q_0^2}{2c}$

$\Rightarrow E = E_i \times e^{-2t_1/Rc}$

From the given data

We know that $E = \frac{E_i}{2}$

$\Rightarrow \frac{E_i}{2} = E_i \times e^{-2t_1/Rc}$

$\frac{1}{2} = e^{-2t_1/Rc}$

$\Rightarrow t_1 = \frac{Rc}{2}\log 2$

take (1) Now $\Rightarrow q = q_0 e^{-t/Rc}$

(given) $\frac{1}{4}q_0 = q_0 e^{-t_2/Rc}$

$\Rightarrow t_2 = 2Rc\ln 2$

$\Rightarrow \frac{t_1}{t_2} = \frac{1}{4}$

37. A thermally insulated rigid container contains an ideal gas. It is heated through a resistance of 100 Ω by passing a current of 1 A for 5 min, then change in internal energy of the gas

A) zero **B)** 30 kJ **C)** 10 kJ **D)** 20 kJ

SOL:

When a current I is flowing through a wire then heat produced is

$\Delta Q = I^2 R t$

Given $R = 100\,\Omega$, $t = 5$ min, $I = 1A$

$\therefore \Delta Q = 1 \times 100 \times 5 \times 60$

$\Delta Q = 30000\,J$

$\Delta Q = 30\,kJ$

$\therefore$ Change in internal energy is 30 kJ.

38. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. Then magneti

A) nB **B)** n^2B **C)** $2\,nB$ **D)** $2\,n^2B$

SOL:

We know that $B = \frac{\mu_0 I}{2r}$

here $r \rightarrow$ radius of circle

Since $l = 2\pi r$

$\Rightarrow r = \frac{l}{2\pi}$

So $B = \frac{\mu_0 I}{2} \times \frac{2\pi}{l} = \frac{\mu_0 I \pi}{l}$ — ①

When $l \rightarrow$ length of wire was bents into loops then $l' = n \times 2\pi r'$

$\Rightarrow B' = \frac{\mu_0 n I}{2r'} = \frac{\mu_0 n I}{2 \frac{l'}{2\pi n}}$

$= \frac{\mu_0 n I}{2} \times \frac{2\pi n}{l'}$

$= n^2 \frac{\mu_0 I \pi}{l'}$

from ① $\boxed{B' = n^2 B}$

39. A circuit contains a resistance of 4 Ω and an inductance of 0.68 H and an alternating effective emf of 500 V of frequency 120 cycles/s. The current is

A) 0.25 A **B)** 0.976 A **C)** 1.5 A **D)** 0.025 A

SOL:

Given $R = 4\,\Omega$, $L = 0.68$ H, $f = 120$ cycles, $V = 500$

we have $Z = \sqrt{R^2 + X_L^2}$

where $X_L = 2\pi f L$

$X_L = 2\pi \times 120 \times 0.68$

$X_L = 512.7\,\Omega$

$\therefore Z = \sqrt{16 + (512.7)^2}$

$Z = 512.72\,\Omega$

So, current $(I) = \frac{500}{512.72} = 0.976\,\Omega$.

40. The electron of a hydrogen atom revolves round the proton in a circular n$^{\text{th}}$ orbit of radius $r_n = \frac{\varepsilon_0 n^2 h^2}{(\pi m e^2)}$ with a speed $v_n = \frac{e^2}{2\varepsilon_0 n h}$. The current due to the circulating charge is propo

A) e^2 **B)** e^3 **C)** e^5 **D)** e^6

SOL:

$i = \frac{e}{t} = ef$ — ①

$V_n = \frac{2\pi r_n}{t}$

$V_n = 2\pi r_n f$

$f = \frac{V_n}{2\pi r_n}$

$f = \frac{e^2}{2\varepsilon_0 n h} \times \frac{\pi m e^2}{2\pi \times \varepsilon_0 n^2 h^2}$

$i = ef$

$i = e \times e^4 = \frac{\pi m}{4\pi \varepsilon_0^2 n^3 h^3}$

$i \propto e^5$

41. The manifestation of band structure in solids is due to

A) Heisenberg's uncertainty principle **B)** Pauli's exclusion principle **C)** Bohr's correspondence principle **D)** Boltzmann's law

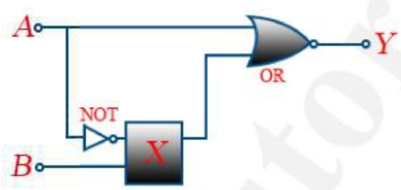
SOL:

According to Pauli's exclusion principle, the electronic configuration of a number of subshells existing in a shell and number of electrons entering each subshell is found.

Hence on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

42. The logic circuit shown in figure yield the following truth table

A	B	Y
1	1	1
0	1	1
1	0	1
0	0	0



The gate X in the diagram is

A) NAND **B)** XOR **C)** AND **D)** NOR

SOL:

The output of the gate is $Y = \bar{A} \cdot B + A$. From truth table

$Y = 1 \Rightarrow \bar{1} \cdot 1 + 1 = 0 \cdot 1 + 1$

$Y = 1 \Rightarrow \bar{0} \cdot 1 + 0 = 1 \cdot 1 + 0$

$Y = 1 \Rightarrow \bar{1} \cdot 0 + 1 = 0 \cdot 0 + 1$

$Y = 0 \Rightarrow \bar{0} \cdot 0 + 0 = 1 \cdot 0 + 0$

The outputs from truth table can be possible if get x is an AND gate.

43. A spring of spring constant $5 \times 10^3 \ \mathrm{Nm}^{-1}$ is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

A) 12.50 Nm **B)** 18.75 Nm **C)** 25.00 Nm **D)** 6.25 Nm

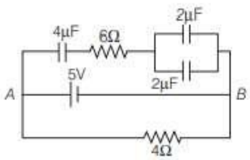
SOL:

$k = 5 \times 10^3$

$x_1 = 5 \text{ cm}, \quad x_2 = 5 + 5 = 10 \text{ cm}$

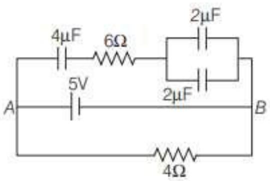
$U = \frac{1}{2} k (x_2^2 - x_1^2) = 18.75 \text{ Nm}.$

44. Calculate the amount of charge on capacitor of $4\mu\mathbf{F}$. The internal resistance of battery is 1Ω.



A) $8\mu\mathrm{C}$ **B)** zero **C)** $16\mu\mathrm{C}$ **D)** $4\mu\mathrm{C}$

SOL: The given circuit diagram is shown below



The battery connected in the circuit is a DC source and capacitor does not allow direct current to pass through it. Therefore, there will no current pass through upper branch of circu

The equivalent resistance of remaining circuit will be

$R_{\text{eq}} = R_{\text{in}} + 4\Omega$

Here, R_{in} is internal resistance of battery.

$R_{\text{eq}} = 1 + 5 = 5\Omega$

From Ohm's law, we know that

$I = \frac{V}{R_{\text{eq}}} = \frac{5}{5} = 1\ \mathrm{A}$

Now, voltage drop across branch AB is calculated as

$V_{\text{AB}} = 1 \times 4 = 4\mathrm{V}$

Voltage across upper branch will also be $4\mathrm{V}$ as the connection is parallel.

Now, charge flowing from upper branch is given as

$\mathrm{Q} = \mathrm{C}_{\text{eq}} \mathrm{V}_{\text{AB}}$ ...(i)

Calculating the equivalent capacitance,

$\frac{1}{C_{\text{eq}}} = \frac{1}{4} + \frac{1}{2+2} = \frac{1}{4} + \frac{1}{4}$

$C_{\text{eq}} = 2$

Substituting the value of equivalent capacitance and voltage drop in Eq. (i), we get

$Q = 2 \times 4\ \mathrm{V} = 8\mu\mathrm{C}$

Therefore, the charge stored in $4\mu\mathbf{F}$ will be $8\mu\mathrm{C}$.

45. A stone is projected up from the top of a tower 58.8 m high with a velocity of 19.6 m/s. It reaches the foot of the tower in:

A) $\sqrt{12}s$ **B)** $\sqrt{6}s$ **C)** $2s$ **D)** $6s$

SOL: If h is the height of the tower,

displacement = −h

So using $\therefore$ $\mathbf{s = ut + \frac{1}{2}at^2}$

$\mathbf{-h = 19.6t - \frac{1}{2} \times 9.8t^2}$

Since u = 19.6 ms^{-1} and it moves up initially.

i.e. −58.8 = 19.6 t − 4.9 t^2

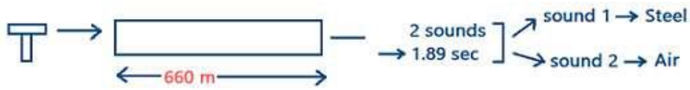
4.9 t^2 − 19.6 t − 4.9 t^2 − 58.8 = t^2 − 4t − 12 = 0

⇒ (t − 6) (t + 2) = 0 ∴ t = 6s

46. One steel pipe of length 660 m is struck a blow which produces loud sound. A listener at the other end hears two sounds at an interval of 1.89 sec; one from the wave that has travell

A) $2.88 \times 10^{11} N/m^2$ **B)** $5.76 \times 10^{11} N/m^2$ **C)** $8.64 \times 10^{11} N/m^2$ **D)** $11.52 \times 10^{11} N/m^2$

SOL:



$P = 8 \times 10^3$ kg/m^3 Also we know,

$V_{air} = 330$ m/s $V_{steel} > V_{air}$

$t_{steel} < t_{air}$

$t_{air} - t_{steel} = 1.89$

$\frac{1}{V_{air}} - t_{steel} = 1.89$

$\frac{660}{330} - t_s = 1.89$

$2 - t_s = 1.89$

$t_s = 0.11$ sec.

$V_{steel} \rightarrow \frac{1}{t}$

$V_s \rightarrow \left(\frac{660}{0.11}\right)$ m/s

$V = \sqrt{\frac{Y}{P}} \rightarrow V_s = \sqrt{\frac{Y_s}{P_s}}$

$\left(\frac{660}{0.11}\right) = \sqrt{\frac{Y_s}{8\times10^3}}$

$Y_s = 288 \times 10^9$

$Y_s = 2.88 \times 10^{11}$ N/m^2

47. If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is

A) 25% **B)** 50% **C)** 100% **D)** 125%

SOL: Given that,

$\frac{\Delta v}{v} \times 100$ is the % error in velocity = 50 %

Kinetic energy K.E = $\frac{1}{2}mv^2$

Error in the kinetic energy

$\frac{\Delta K.E}{K.E} \times 100 = m \times 2\frac{\Delta v}{v} \times 100$

m is as a constant

Now, percentage error

$\frac{\Delta K.E}{K.E} \times 100 = 2\frac{\Delta v}{v} \times 100$

$\frac{\Delta K.E}{K.E} \times 100 = 2 \times 50\%$

$\frac{\Delta K.E}{K.E} \times 100 = 100\%$

Hence, the error in the measurement of kinetic energy is 100 %

48. The mass of an α-particle is

A) Less than the sum of masses of two protons and two neutrons **B)** Equal to mass of four protons **C)** Equal to mass of four neutrons **D)** Equal to sum of masses of two protc

SOL: This is due to mass defect because a part of mass is used in keeping the neutrons and protons bound as α-particle.

49. The value of $\frac{\sin\left(\frac{-11\pi}{3}\right)\cdot\tan\left(\frac{35\pi}{6}\right)\cdot\sec\left(\frac{-7\pi}{3}\right)}{\cos\left(\frac{5\pi}{4}\right)\cdot\text{cosec}\left(\frac{7\pi}{4}\right)\cdot\cos\left(\frac{17\pi}{6}\right)}$ is

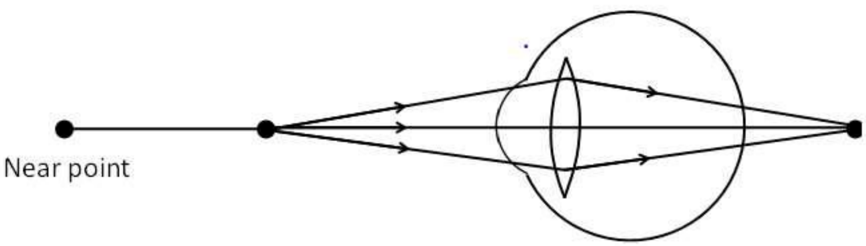
A) $\frac{15}{\sqrt{3}}$ **B)** $\frac{2}{\sqrt{3}}$ **C)** $\frac{8}{\sqrt{3}}$ **D)** $\frac{1}{\sqrt{3}}$

SOL: Required Solution

50. Which of the following lenses is used to correct the defect of hypermetropia?

A) Bifocal lens **B)** Convex lens **C)** Concave lens **D)** Cylindrical lens

SOL: Hypermetropia is a defect of eye in which a person is able to see distant objects but unable to see nearby objects clearly. Hypermetropia occurs when either the focal length of the ey



This defect of eye is corrected by the use of convex lens. The convex lens converges all the incoming rays at the retina of the eye as represented in the given figure.

51. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. '

SOL:

Smallest main scale division = 0.05 cm (5.15 - 5.10)

M.S.R = 5.10 cm

VC = 24

L.C = 0.05 - 2.45 / 50 = 0.001 cm

Diameter = MSR + VC x LC

= 5.10 + 24 x 0.001 = 5.124 cm

52. A quarter horse power motor runs at a speed of 600 r.p.m. Assuming 40% efficiency the work done(in J) by the motor in one rotation will be______.

SOL: Motor makes 600 revolution per minute

$\therefore n = 600\frac{\text{revolution}}{\text{minute}} = 10\frac{\text{rev}}{\text{sec}}$

$\therefore$ Time required for one revolution = $\frac{1}{10}$ sec

Energy required for one revolution = power × time

$= \frac{1}{4} \times 746 \times \frac{1}{10} = \frac{746}{40}$ J

But work done = 40% of input

$= 40\% \times \frac{746}{40} = \frac{40}{100} \times \frac{746}{40} = 7.46$ J

53. A rubber tube of length 8 m is hung from the ceiling of a room.then the increase in length of the rope due to its own weight is (in mm)___________. (Given Young's modulus of ela

SOL: As, $Y = \frac{Mg}{A} \times \frac{L/2}{\Delta L}$ [length is taken as $\frac{L}{2}$ because weight acts at centre of gravity (CG)]

Now, M = ALρ (In order to calculate the mass, the whole of geometrical length L is to be considered)

$\therefore Y = \frac{AL\rho gL}{2A\Delta L} \Rightarrow \Delta L = \frac{\rho gL^2}{2Y} = \frac{1.5\times10^3\times10\times8\times8}{2\times5\times10^6}$

$= 9.6 \times 10^{-2}$ m $= 9.6 \times 10^{-2} \times 10^3$ mm = 96 mm

54. The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21%. The fundamental frequency of the sonometer wire in a Hz is

SOL: Frequency ∝ (Tension)1/2

$\frac{\Delta n}{n} \times 100 = \left[\left(\frac{121}{100}\right)^{1/2} - 1\right] \times 100$

$\frac{5}{n} \times 100 = \left[\frac{11}{10} - 1\right] \times 100$

n = 50

Fundamental frequency of sonometer = 50 Hz

55. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be_____.

SOL:

$R = 12\,\Omega \quad Z = 15$

$X_L = \sqrt{(15)^2 - (12)^2} = \sqrt{225 - 144}$

$= \sqrt{81} = 9$

$\tan\phi = \frac{X_L}{R} = \frac{9}{12} = 0.75$

$\phi = \tan^{-1}(0.75)$

$\Rightarrow \cos\phi = \cos(\tan^{-1}(0.75))$

$= \underline{0.8}$

56. Sea water at frequency $v = 4 \times 10^8$ Hz has permittivity $\varepsilon = 80\,\varepsilon_0$, permeability $\mu \approx \mu_0$ and resistivity $\rho = 0.25$ Ω–m. Imagine a parallel plate capacitor immersed in sea water and dri

SOL: $V(t) = V_o \sin 2\pi vt$

Let distance between the plates = d.

Electric field = $\frac{V(t)}{d} = \frac{V_o}{d} \sin 2\pi vt$

Conduction current density

$J_c = \frac{E}{\rho} = \frac{V_o}{\rho d} \sin 2\pi\upsilon t = J_{o_c} \sin 2\pi\upsilon t$

where J_{o_c} is maximum conduction current density. Displacement current density,

$J_d = \varepsilon \frac{d}{dt}\left[\frac{V_o}{d}\sin(2\pi\upsilon t)\right] = \frac{2\pi\upsilon\varepsilon}{d} V_o \cos 2\pi\upsilon t$

$J_d = J_{o_d} \cos(2\pi\upsilon t)$

$\frac{J_{o_d}}{J_{o_c}} = \frac{2\pi\upsilon}{V_o}\varepsilon\frac{V_o}{d} = 2\pi\upsilon\varepsilon\rho = 2\pi(4 \times 10^8)(80\varepsilon_0)(0.25)$

$= \frac{J_{o_c}}{J_{o_d}} = \frac{9\times10^9}{4\times10^9} = \frac{9}{4} \quad \left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{N m}^2/\text{C}^2\right)$

57. Two thin symmetrical lenses of different nature and of different material have equal radii of curvature R = 60 cm. The lenses are put close together and immersed in water $\left(\mu_\omega = \frac{4}{3}\right.$

SOL: Let f_1 and f_2 be the focal length of lenses in water. Then

$\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_\omega} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$

$\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_\omega} - 1\right)\left(\frac{2}{R}\right)$ ...(1)

and $\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_\omega} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$

$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_\omega} - 1\right)\left(-\frac{2}{R}\right)$ ...(2)

Adding equation (1) & (2) we get

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2(\mu_1 - \mu_2)}{\mu_\omega R}$

$\frac{1}{30} = \frac{2(\mu_1 - \mu_2)}{\mu_\omega R}$

$(\mu_1 - \mu_2) = \frac{4}{3}\frac{R}{60} = \frac{4}{3} \times \frac{60\ \text{cm}}{60\ \text{cm}} = \frac{4}{3} = 1.333$

58. An interference pattern is produced using Young's double slit experiment with sunlight as source. Assume the wavelength of blue light and red light roughly a 400 nm and 600 nm, r

SOL:

Given $\lambda_r = 600nm$, $\lambda_b = 400\ nm$.

Let the nth red fringe coincide with $(n+1)^{th}$ blue fringe.

Then, the distance from the centre for the n^{th} red fringe in $n \times \beta_r$.

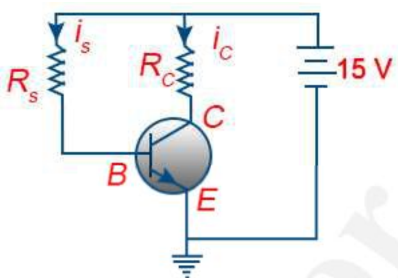
But $\beta \propto \lambda$. Hence $\beta_r = n\lambda r$

Similarly for $(n+1)^{th}$ blue fringe $\beta_b = (n+1)\lambda_b$

When they are at equal distance from centre then

$$n\lambda_r = (n+1)\lambda_b$$
$$n(6000) = (n+1)(4000)$$
$$6000n = 4000x + 4000$$
$$2000n = 4000 \Rightarrow \boxed{n=2}$$

59. In the following common emitter circuit if β = 100, $V_{CE} = 7V$, V_{BE} = negligible, R_C = 2 kΩ then I_B is_____mA.



SOL:

$\beta = 100$, $V_{CE} = 7V$

$R_C = 2k\Omega$, $V_{BE} = 0$

$V_{CC} = V_{CE} + i_C R_C$

$15 = 7 + P_C \times 2 \times 10^3$

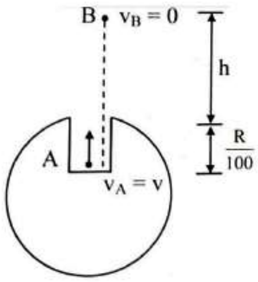
$i_C = 4 \times 10^{-3} A$

$\beta = \frac{i_C}{i_B} \Rightarrow i_B = \frac{i_C}{\beta} = \frac{4}{100}$ mA

$i_B = 0.04$ mA

60. There is a crater of depth $\frac{R}{100}$ on the surface of the moon (radius $\mathbf{R}$). A projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity ' $\mathbf{v}$ ' fi

SOL:



Speed of particle at A,

$$v_A = v = \sqrt{\frac{2GM}{R}}$$

At point B, $v_B = 0$

By conservation of energy,

decrease in KE = increase in GPE

$$\Rightarrow \frac{1}{2}Mv_A^2 = U_B - U_A$$

$$\therefore \frac{1}{2}M\cdot\left(\frac{2GM}{R}\right) = M(V_B - V_A)$$

$$\therefore \frac{GM}{R} = -\frac{GM}{R+h} - \left[\frac{-GM}{2R}\left[3 - \frac{\left(R - \frac{R}{100}\right)^2}{R^2}\right]\right]$$

$$\therefore \frac{1}{R} = -\frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^2 \cdot \frac{1}{R}$$

$$\therefore \frac{1}{R} = -\frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2}\right)\left(\frac{0.98}{R}\right)$$

$$\therefore \frac{1}{R+h} = \frac{1}{2R}[1 - 0.98]$$

$$\therefore 2R = 0.02(R+h)$$

$$\therefore h = \frac{1.98}{0.02}$$

$$\therefore h = 99\,R$$

61. The oxide of an element contains 67.67 % of oxygen and the vapour density of its volatile chloride is 79. Equivalent weight of the element is

A) 2.46 **B)** 3.82 **C)** 4.36 **D)** 4.96

SOL:

100g of oxide will contain 67.67 g of oxygen
32.33 g of element

67.67g of oxygen combines with 32.33g of element

1g of oxygen – ?

$= \frac{32.33}{67.67}$

Now 8g of oxygen — ?

$= \frac{32.33}{67.67} \times 8$

$= 3.82$ g eq wt

62. $l = 3$, then the values of magnetic quantum numbers are

A) ± 1, ± 2, ± 3 **B)** 0, ± 1, ± 2, ± 3 **C)** –1, –2, –3 **D)** 0, + 1, + 2, + 3

SOL:

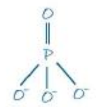
for $l = 3$

$m = -3, -2, -1, 0, 1, 2, 3$

63. In PO_4^{3-} ion the formal charge on the oxygen atom of P—O bond is

A) +1 **B)** −1 **C)** −0.75 **D)** +0.75

SOL:



Formal charge of O can be calculated as

formal charge on O atom of p-O bond $= 6 - 6 - \frac{1}{2}(2) = -1$

64. 0.16 g of methane is subjected to combustion at 27°C in a bomb calorimeter system. The temperature of the calorimeter system (including water) was found to rise by 0.5°C. Calcu

A) $-695\ kJ\ mol^{-1}$ **B)** $-1703\ kJ\ mol^{-1}$ **C)** $-890\ kJ\ mol^{-1}$ **D)** $-885\ kJ\ mol^{-1}$

SOL:

Given

wt. of methane = 0.16g

Heat produced = 17.7 x 0.5

= 8.85 KJ

Now, heat produced by combustion of 1.0 mole of CH_4 $= \frac{8.85 \times 16}{0.16}$

= 885 KJ

Since the combustion takes place in a closed vessel, the heat produced is

q = −885 KJ

65. 18 ml of mixture of acetic acid and sodium acetate required 6 ml of 0.1 M NaOH for neutralization of the acid and 12 ml of 0.1 M HCl for reaction with salt separately. If pK_a of the

A) 5.05 **B)** 4.75 **C)** 4.5 **D)** 4.6

SOL:

Given $V_1 = 6ml$ | $V_2 = 12ml$

$M_1 = 0.1m$ | $M_2 = 0.1m$

$pK_a = 4.75$

concentration of acid = 6 x 0.1

= 0.6

concentration of salt = 12 x 0.1

= 1.2

we know that, $pH = pK_a + \log\frac{[salt]}{[acid]}$

$= 4.75 + \log\left(\frac{1.2}{0.6}\right)$

$= 4.75 + \log(2)$

$= 4.75 + 0.3010$

$\therefore pH = 5.05$

66. The oxidation state of oxygen in $O_2\ PtF_6$ is

A) zero **B)** − ½ **C)** + ½ **D)** +1

SOL:

Oxidation state of oxygen in O_2PtF_6, is +1/2. There is structure O_2PtF_6 has O_2 unit and PtF_6 is strong oxidising agent So, O_2 has +1/2 oxidation state.

67. The oxidation state of boron family shows which of the following trend for stable +1 oxidation state?

A) Al < Ga < In < Tl (+1 O.S. stability increases) **B)** Al < Ga < In < Tl (stability of +3 oxidation states) **C)** Al < Ga < In < Tl (stability of +1 oxidation states) **D)** Al < Ga < I

SOL:

The general oxidation states of boron family are +3 and +1. When the element is in +1 oxidation state, it has more stability when compared to +3 oxidation state. The stability increases down the group.

68. The percentage of sulphur in an organic compound whose amount of 0.32 g produces 0.233 g of $BaSO_4$ (Atomic weight of Ba = 137, S = 32) is

A) 1.0 **B)** 10.0 **C)** 25.3 **D)** 32.1

SOL:

Given

wt. of sulphur = 0.32g

wt. of $BaSO_4$ = 0.233g

we know that

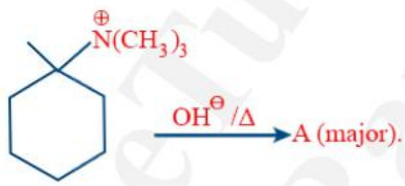
Mol. wt. of $BaSO_4$ = 137 + 32 + 4(16)

= 233g

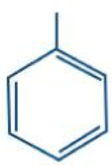
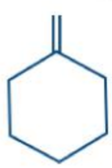
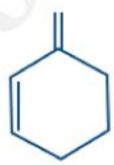
$\therefore$ % of sulphur = $\frac{\text{at. wt of sulphur}}{\text{mol. wt of } BaSO_4} \times \frac{\text{wt. of } BaSO_4}{\text{wt. of sulphur}} \times 100$

$= \frac{32}{233} \times \frac{0.233}{0.32} \times 100$

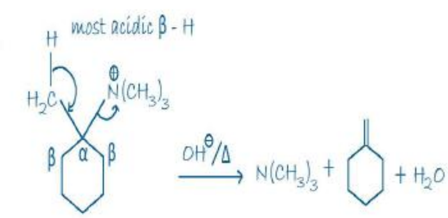
= 10 %

69. 

The major organic product 'A' is

A)  **B)**  **C)**  **D)** 

SOL:



70. An ideal solution contains two volatile liquids A (P^0 = 100 torr) and B (P^0 = 200 torr). If mixture contains 1 mole of 'A' and 3 moles of 'B'. Then, total vapour pressures of distillate

A) 150 torr **B)** 188.8 torr **C)** 185.72 torr **D)** 198.88 torr

SOL:

$$P = p^{\circ}_A x_A + p^{\circ}_B x_B$$
$$= 100 \times \frac{1}{4} + 200 \times \frac{3}{4} = 25 + 150 = 175$$
$$P_{distillate} = 100 \times \frac{25}{175} + 200 \times \frac{150}{175}$$
$$= 125.72 \text{ torr.}$$

71. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100Ω. The conductivity of this solution is 1.29 S m^{-1}. Resistance of the same cell v

A) 124×10^{-4} S m^2 mol^{-1} **B)** 1240×10^{-4} S m^2 mol^{-1} **C)** 1.24×10^{-4} S m^2 mol^{-1} **D)** 12.4×10^{2} S m^2 mol^{-1}

SOL:

$$\frac{1}{R} \cdot \frac{l}{a} = \frac{1}{e}$$
$$\frac{1}{100} \frac{l}{a} = 1.29$$
$$\frac{l}{a} = 1.29 \times 100$$
$$\frac{1}{520} \cdot 1.29 \times 100 = K$$
$$K = 2.48 \times 10^{-1}$$
$$\Lambda_m = \frac{2.48 \times 10^{-1} \times 1000}{0.2}$$
$$= 12.4 \times 10^{2} \text{ S m}^2 \text{ mol}^{-1}$$

72. The reaction $A \rightarrow B$ follows first order kinetics. The time taken for 0.8 mole of A to produce 0.6 mole of is 1 hour. What is the time taken for conversion of 0.9 mole of A to produce

A) 2 hour **B)** 1 hour **C)** 0.5 hour **D)** 0.25 hour

SOL:

$$K = \frac{2.303}{t} \log \frac{[R]_0}{[R]}$$
$$K = \frac{2.303}{1} \log \frac{0.8}{0.6}$$
$$= \frac{2.303}{1} \log (1.3333)$$
$$K = 2.303 \times 0.1249$$
$$2.303 \times 0.1249 = \frac{2.303}{t} \log \frac{0.9}{0.675}$$
$$t = \frac{1}{0.1249} \log (1.3333)$$
$$= \frac{1}{0.1249} \times 0.1249$$
$$t = 1 \text{ hr}$$

73. The chief source of iodine in which it is present as sodium iodate is

A) Sea weeds **B)** Caliche **C)** Carnallite **D)** Iodine never exists as sodium iodate

SOL:

The chief source of iodine in which it is present as sodium iodate is Caliche. $NaIO_3$.

74. What is the correct order of spin only magnetic moment (in B.M.) of Mn^{2+} , Cr^{2+} and V^{2+}?

A) $Mn^{2+} > V^{2+} > Cr^{2+}$ **B)** $V^{2+} > Cr^{2+} > Mn^{2+}$ **C)** $Mn^{2+} > Cr^{3+} > V^{2+}$ **D)** $Cr^{2+} > V^{2+} > Mn^{2+}$

SOL:

The correct order of spin only magnetic moment (in B.M) of Mn^{+2}, Cr^{+2} and V^{+2} is

$Mn^{+2} > Cr^{+2} > V^{+2}$

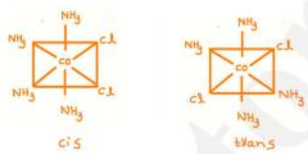
75. $[CO(NH_3)_4(NO_2)_2]Cl$ exhibits

A) linkage isomerism, ionisation isomerism and optical isomerism **B)** linkage isoerism, geometrical isomerism and optical isomerism **C)** linkage isomerism, ionisation isomeri

SOL:

$[Co(NH_3)_4(NO_2)_2]Cl$ and $[Co(NH_3)_4NO_2Cl]NO_2$ → Ionisation isomerism

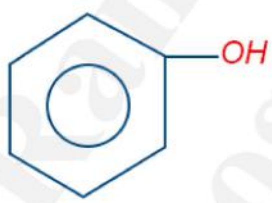
$[Co(NH_3)_4(NO_2)_2]Cl$ and $[Co(NH_3)_4(ONO)_2]Cl$ → linkage isomerism



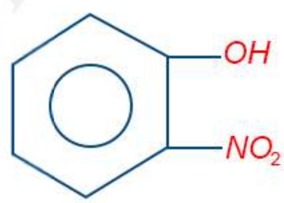
geometrical isomerism

76. Which of the following compounds is most acidic

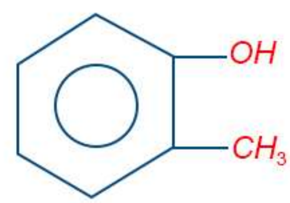
A) $Cl-CH_2-CH_2-OH$



B)



C)

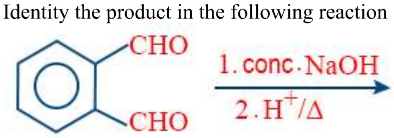


D)

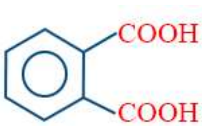
SOL:

(o-nitrophenol, $-OH$, $-NO_2$) is most acidic due to $-NO_2$ is a electron withdrawing group. so $-O-H$ bond is weak so easily H^+ ion released.

77. Identity the product in the following reaction :

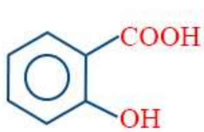


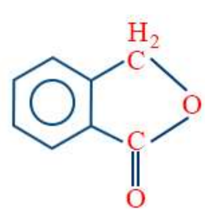
A)



B)

C)

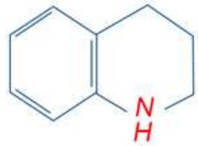
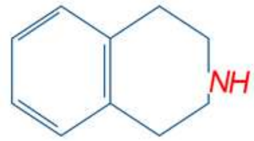
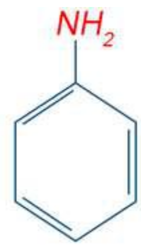




D)

SOL: This is an example of intramolecular cannizzaro reaction.

78. Which of the following is most basic?

A)  **B)**  **C)**  **D)** 

SOL:

Among the given compounds,  is most basic because the lone pair of electrons in the compound does not involve in delocalization.

79. Which of the following is true?
(i) sucrose is a non reducing agent
(ii) glucose is oxidized by bromine water
(iii) glucose rotates plane polarized light in clockwise direction
(iv) fructose is oxidized by bromine water
Select the correct answer during the coded given below

A) (i), (ii), (iii) **B)** (i), (ii) only **C)** (ii), (iii) only **D)** (i) (iv) only

SOL:

(i) sucrose is a non reducing agent
(ii) glucose is oxidized by bromine water
(iii) glucose rotates plane polarized light in clockwise direction.

80. The first ionisation potential of Na, Mg, Al and Si are in order

A) Na > Mg > Al > Si **B)** Na < Mg < Al < Si **C)** Na < Si < Al < Mg **D)** Na < Al < Mg < Si

SOL:

Ionization potential increases from left to right in the periodic table. since Mg has fully filled s-orbital, it has more stability than Al. so, the order of first ionization energy of Na, Mg, Al and Si is

Na < Al < Mg < Si

81. If equal moles of water and urea are taken in a vessel. ___________ will be the mass percentage of urea in the solution.

SOL:

$$\%\ \text{weight of urea} = \frac{\text{wt. of urea}}{\text{wt. of urea} + H_2O} \times 100$$

$$= \frac{18}{78} \times 100$$

$$= 23.076\ \%$$

82. Let IP stand for ionization potential. The IP, and IP_2 of Mg are 178 and 348 kcal mol^{-1}. The energy required for the following reaction is _________ kcal.

$Mg \rightarrow Mg^{2+} + 2e^{-}$

SOL:

$$Mg(g) \longrightarrow Mg^{+}(g) + e^{-}\ ;\ I.E_1 = 178\ \text{Kcal/mol}$$

$$Mg^{+}(g) \longrightarrow Mg^{+2}(g) + e^{-}\ ;\ I.E_2 = 348\ \text{Kcal/mol}$$

$$Mg(g) \longrightarrow Mg^{+2}(g) + 2e^{-}\ ;\ I.E = I.E_1 + IE_2 = +526\ \text{Kcal/mol}$$

83. O_2^{2+} has a bond order is _______.

SOL:

O_2^{+2}

$$B.O = \frac{1}{2}[e^{-}.B.O - e^{-}.\text{Anti } B.O]$$

$$= \frac{1}{2}[10-4]$$

$$= \frac{6}{2}$$

$$= 3$$

84. The number of isomers possible for C_7H_8O are _______

SOL:

Given compound is C_7H_8O.
The possible number of isomers for C_7H_8O are 5. They are benzyl alcohol, anisole, o-, m- and p- cresols (methyl phenols).

85. The charge required to deposit 40.5 g of Al (atomic mass = 27.0 g) from the fused $Al_2(SO_4)_3$ is _________ $\times 10^5$ C.

SOL:

$$Al_2(SO_4)_3 \longrightarrow 2Al^{+3} + 3SO_4^{-2}$$

27g of Al requires 96500×3 c

40.5g of Al requires ?

$$= \frac{40.5 \times 96500 \times 3}{27}$$

$$= 434250\ c$$

$$= 4.34 \times 10^{5}\ c$$

86. A reaction takes place in three steps the rate constant are K_1, K_2 and K_3 the overall rate constant $K = \frac{K_1K_3}{K_2}$. If the energies of activation are 40, 30 and 20 kJ/mol, the overall energy

SOL: $K = \frac{K_1K_3}{K_2}$

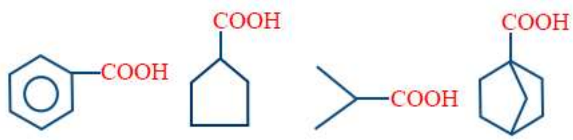
$\therefore K = A.e^{-\frac{Ea}{RT}}$ $[\therefore Ea = 30\ kJ/mol^{-1}]$

$\therefore K = A.e^{-(Ea_1 + Ea_3 - Ea_2)/RT}$

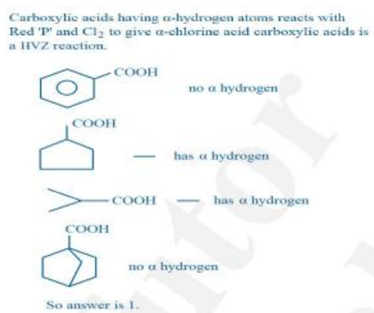
$= A.e^{-(40 - 30 + 20)/RT}$

$= A.e^{-30/RT}$

87. How many the following compounds will give HVZ reaction



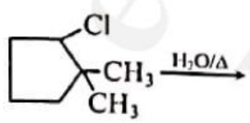
SOL:



88. Find the oxidation no of Mn in the product of alkaline oxidative fusion of MnO_2 ?

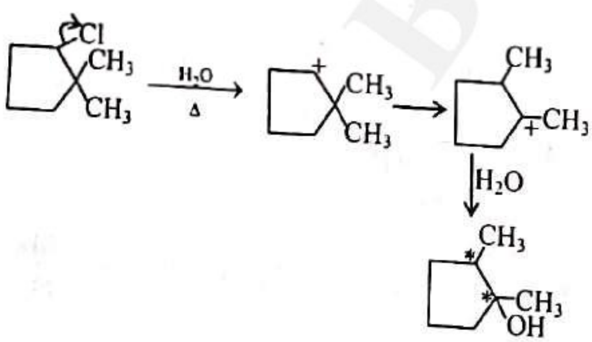
SOL: $2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$

89.



How many 3° alcohol are obtain as product (including stereoisomer)?

SOL:



S.I. $= 2^2 = 4$

90. How many among the following are oligosaccharides?
i. Cellulose
ii. Lactose
iii. Sucrose
iv. Raffinose
v. Ribose
vi. Galactose
vii. Glycogen
viii. Starch

SOL: Carbohydrates that yield two to ten monosaccharide units, on hydrolysis, are called oligosaccharides. Lactose (disaccharide), sucrose (disaccharide) and raffinose (trisaccharide) are

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