SECTION NAME

Date : 06/03/2024 **MOCK TEST - 1**
Time : 3 Hours 0 Minutes **Marks : 300**

Mathematics

1. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is

A) $a\cot\left(\frac{\pi}{n}\right)$ **B)** $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ **C)** $a\cot\left(\frac{\pi}{2n}\right)$ **D)** $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

SOL:

$R + r$

$= \frac{a}{2}\operatorname{Cosec}\left(\frac{\pi}{n}\right) + \frac{a}{2}\operatorname{Cot}\left(\frac{\pi}{n}\right)$

$= \frac{a}{2}\left[\frac{1}{\operatorname{Sin}\left(\frac{\pi}{n}\right)} + \frac{\operatorname{Cos}\left(\frac{\pi}{n}\right)}{\operatorname{Sin}\left(\frac{\pi}{n}\right)}\right]$

$= \frac{a}{2}\left[\frac{1 + \operatorname{Cos}\left(\frac{\pi}{n}\right)}{\operatorname{Sin}\left(\frac{\pi}{n}\right)}\right]$

$= \frac{a}{2}\,\frac{2\operatorname{Cos}^2\left(\frac{\pi}{2n}\right)}{2\operatorname{Sin}\left(\frac{\pi}{2n}\right)\operatorname{Cos}\left(\frac{\pi}{2n}\right)}$

$= \frac{a}{2}\operatorname{Cot}\left(\frac{\pi}{2n}\right)$

2. If α, β be the roots of $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$, $q + r \neq 0$, then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$

A) 1 **B)** q **C)** r **D)** $q + r$

SOL:

Given α, β roots of $x^2+px-q=0$, γ, δ be the roots of $x^2+px+r=0$

$\therefore \alpha+\beta = -p \Rightarrow \alpha+\beta = \gamma+\delta$, $\gamma+\delta = -p$

$\alpha\beta = -q$

Similarly, $\gamma+\delta = -p$

$\gamma\delta = r$

Now $(\alpha-\gamma)(\alpha-\delta) = \alpha^2 - \alpha\delta - \gamma\alpha + \gamma\delta$

$= \alpha^2 - \alpha\delta - \gamma\alpha + r$

$= \alpha^2 - \alpha(\delta+\gamma) + r$

$= \alpha^2 - \alpha(\alpha+\beta) + r$

$= \alpha^2 - \alpha^2 - \alpha\beta + r$

$= -\alpha\beta + r$

$= -(-q) + r$

$= q + r$

By Symmetry will growth $(\beta-\gamma)(\beta-\delta)$

$\therefore (\beta-\gamma)(\beta-\delta) = q + r$

$\therefore$ Ratio is 1.

3. In the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$, the term independent of x is

A) 10^{th} **B)** 9^{th} **C)** 8^{th} **D)** 7^{th}

SOL: For $(x+a)^n$, $T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot a^r$

for $\left(2x^2 - \frac{1}{x}\right)^{12}$, $T_{r+1} = {}^{12}C_r\left(2x^2\right)^{12-r} \cdot \left(\frac{-1}{x}\right)^r$

$T_{r+1} = {}^{12}C_r \cdot 2^{12-r} \cdot x^{2(12-r)} (-1)^r \cdot x^{-r}$

$= {}^{12}C_r \cdot (-1)^r \cdot 2^{12-r} \cdot x^{24-3r}$

To get independent term $x^0 = x^{24-3r}$

$24-3r=0$

$r = \frac{24}{3} = 8$

$r=8$

∴ Term independent of (i.e, x^0) $= T_{r+1}$

$= T_{8+1} = T_9$

∴ 9^{th} term is independent of 'x'.

4. Suppose a, b, c are distinct real numbers. If a, b, c are in A.P and a^2, b^2, c^2 are in H.P., then

A) $-\frac{a}{2}$, b, c are in G.P. **B)** $a+b=c$ **C)** $a=b+c$ **D)** a, b, c are in G.P.

SOL: $2b = a + c$, $b^2 = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow \frac{(a+c)^2}{4} = \frac{2a^2c^2}{a^2+c^2}$

$\Rightarrow (a^2+c^2)^2 + 2ac(a^2+c^2) - 8a^2c^2 = 0$

$\Rightarrow (a^2+c^2+4ac)(a^2+c^2-2ac) = 0$

$\Rightarrow [(a+c)^2 + 2ac](a-c)^2 = 0$

$\Rightarrow (a+c)^2 + 2ac = 0$ $[\because a \neq c]$

$\Rightarrow 4b^2 + 2ac = 0$

$\Rightarrow -\frac{a}{2}$, b, c are in G.P.

5. If points (5, 5), (10, k) and (−5, 1) are collinear, then $k =$

A) 3 **B)** 5 **C)** 7 **D)** 9

SOL: Let the given points be $A(5,5)$ $B(10,k)$, $C(-5,1)$

If the above points are collinear.

they will lie on the same line.

(i.e) The will not from triangle.

$\therefore$ Area of $\Delta ABC = 0$

$\Rightarrow \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=0$

$\therefore A(5,5),\ B(10,k),\ C(-5,1)$

$\Rightarrow \frac{1}{2}\left[5(k-1)+10(1-5)+(-5)(5-k)\right]=0$

$\Rightarrow \frac{1}{2}\left[5(k-1)-40-25+5k\right]=0$

$\Rightarrow \frac{1}{2}\left[5k-5-40-25+5k\right]=0$

$\Rightarrow \frac{1}{2}\left[10k-70\right]=0$

$\Rightarrow 10k-70=0$

$k=7$

6. The eccentricity of an ellipse whose pair of a conjugate diameter are $y = x$ and $3y = -2x$ is

A) $\frac{2}{3}$ **B)** $\frac{1}{3}$ **C)** $\frac{1}{\sqrt{3}}$ **D)** None of these

SOL: Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ — ①

$\therefore y=x$ and $3y=-2x$ is a pair of conjugate diameter.

$\therefore m_1 m_2 = -\frac{b^2}{a^2}$ (from ①)

$\Rightarrow 1\cdot\left(-\frac{2}{3}\right) = -\frac{b^2}{a^2}$ $\left[\because \frac{y}{x}=1=m_1,\ \frac{y}{x}=-\frac{2}{3}=m_2\right]$

$\Rightarrow 2a^2 = 3b^2$

$\Rightarrow 2a^2 = 3a^2(1-e^2)$

$\Rightarrow e^2 = \frac{1}{3}$

$e = \frac{1}{\sqrt{3}}$ [$\because e>0$ always]

7. The value of $\lim_{x\to 1} \frac{x^n+x^{n-1}+x^{n-2}+....+x^2+x-n}{x-1}$ is

A) $\frac{n(n+1)}{2}$ **B)** 0 **C)** 1 **D)** n

SOL:

$$\lim_{x\to 1} \frac{x^n+x^{n-1}+x^{n-2}+\dots+x^2+x-x}{x-1}$$

$$= \lim_{x\to 1} \frac{nx^{n-1}+(n-1)x^{n-2}+(n-2)x^{n-3}+\dots+2x+1}{1}$$

$$= n+(n-1)+(n-2)+\dots+2+1$$

$$= \frac{n(n+1)}{2}$$

8. A sample of 35 observations has the mean 80 and standard deviation as 4. A second sample of 65 observation from the same population has mean 70 and standard deviation 3, then the standard deviation of the combined sample is

A) 5.85 **B)** 5.58 **C)** 34.2 **D)** None of these

SOL:

$n_1 = 35,\ n_2 = 65,\ \bar{x}_1 = 80,\ \bar{x}_2 = 70$

$\sigma_1 = 4 \quad \sigma_2 = 3$

Combined standard deviation

$$= \sqrt{\frac{35(16 + 42 \times 25) + 65(9 + 12 \times 25)}{35 + 65}}$$

$$= \sqrt{34.21}$$

$$= 5.85$$

9. Let R be the real line. Consider the following subjects of the plane $R \times R$.

$S = [(x, y) : y = x + 1 \text{ and } 0 < x < 2]$, $T = [(x, y) : x - y \text{ is an integer}]$. Which one of the following is true?

A) Neither S nor T is an equivalence relation on R **B)** Both S and T are equivalence relations on R

C) S is an equivalence relation on R but T is not **D)** T is an equivalence relation on R but S is not

SOL:

$T : [(x, y)\ x - y$ is integer$]$

if $x = y$ then $x - x = 0$ also integer it is reflexive $(y - x)$ is also integer it is symmetric for some z.

$(x - z)$ and $(z - x)$ are integers it is transitive

$\therefore$ T is Equivalence relation

$S = \{y = x + 1,\ 0 < x < 2\}$

$y - x = 1$ so if $x = y \rightarrow 0 = 1$ not possible

it is not reflexive

so, T is equivalence but not 'S'.

10. The number of solutions to the equation $\tan^{-1}\left(\frac{x}{3}\right) + \tan^{-1}\left(\frac{x}{2}\right) = \tan^{-1} x$ is

A) 3 **B)** 2 **C)** 1 **D)** 0

SOL:

$\tan^{-1}\left(\frac{x}{3}\right) + \tan^{-1}\left(\frac{x}{2}\right) = \tan^{-1}x$

$\tan^{-1}\left[\frac{x/3 + x/2}{1 - x^2/6}\right] = \tan^{-1}x$

where $x>0$ and $\frac{x^2}{6} < 1 \Rightarrow -\sqrt{6} < x < \sqrt{6}$

Now, $\left(\frac{5x}{6-x^2}\right) = x$

$\Rightarrow x=0$, or $x^2-1=0 \Rightarrow x = \pm 1$,

There fore, $x = \{-1, 0, 1\}$

$\therefore$ 3 solution

11. The value of θ in $[0, 2\pi]$ such that the matrix

$$\begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\theta+\pi) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\theta-\pi) & \tan(\pi \boxtimes \theta) & 0 \end{bmatrix}$$

is skew-symmetric, is

A) $\pi/2$ **B)** $\pi/3$ **C)** $\pi/4$ **D)** $\pi/6$

SOL:

let $A = \begin{bmatrix} 2\sin\theta-1 & \sin\theta & \cos\theta \\ -\sin\theta & 2\cos\theta-\sqrt{3} & \tan\theta \\ -\cos\theta & -\tan\theta & 0 \end{bmatrix}$, $A^T = \begin{bmatrix} 2\sin\theta-1 & -\sin\theta & -\cos\theta \\ \sin\theta & 2\cos\theta-\sqrt{3} & -\tan\theta \\ \cos\theta & \tan\theta & 0 \end{bmatrix}$

Since, A is skew symmetric $A^T = -A$

$\Rightarrow 2\sin\theta - 1 = -(2\sin\theta - 1) \Rightarrow 4\sin\theta = 2$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

12. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $A^{-1}\frac{1}{6} = \left(A^2 + CA + DI\right)$ then C and D are equal to

A) $-11, 6$ **B)** $-6, 31/6$ **C)** $6, 11$ **D)** $-6, -11$

SOL:

Given

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\frac{A^{-1}}{6} = A^2 + CA + DI \Rightarrow C, D = ?$

$|A| = 1(4+2) + 0 + 0$
$= 8 + 0$

$\therefore$ A is invertible.

$AA^{-1} = A^{-1}A = I$

multiply both sides by A.

$\frac{AA^{-1}}{6} = A^3 + CA^2 + DAI$

$\Rightarrow \frac{1}{6}I = A^3 + CA^2 + DAI$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix}$

$\frac{1}{6}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -11 & 19 \\ 0 & -38 & 46 \end{bmatrix} + C\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + D\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$

comparing $1 + C + D = \frac{1}{6} \rightarrow ①$

$-11 - C + D = 1/6 \rightarrow ②$

Add ①&②

$-10 + 2D = \frac{2}{6} = \frac{1}{3}$

$2D = \frac{1}{3} + 10$

$2D = \frac{31}{3}$

$D = 31/6$

substitute $D = \frac{31}{6}$ in ①

$1 + C + \frac{31}{6} = \frac{1}{6}$

$1 + C = \frac{1}{6} - \frac{31}{6} = -\frac{30}{6} = -5$

$1 + C = -5$

$C = -6$

$\therefore C = -6,\ D = \frac{31}{6}$

13. If $\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\cos\left(\frac{x}{2^3}\right)\ldots\ldots\ldots\infty = \frac{\sin x}{x}$ then $\frac{1}{2^2}\sec^2\left(\frac{x}{2}\right) + \frac{1}{2^4}\sec^2\left(\frac{x}{2^2}\right) + \ldots\ldots\infty =$________

A) $\mathrm{cosec}^2 x - \frac{1}{x}$ **B)** $\mathrm{cosec}^2 x - \frac{1}{x^2}$ **C)** $\mathrm{cosec}^2 x + \frac{1}{x}$ **D)** $\mathrm{cosec}^2 x + \frac{1}{x^2}$

SOL:

$\cos x/2 \cos(x/2^2)\cos(x/2^3) \ldots \infty = \frac{\sin x}{x}$

Apply 'log' on both sides

$\log \cos\frac{x}{2} + \log(\cos x/2^2) + \ldots \infty = \log\left(\frac{\sin x}{x}\right)$

Apply differentiation

$-\left[\frac{1}{2}\tan x/2 + 1/2^2 \cdot \tan\left(\frac{x}{2^2}\right) + \ldots \infty\right] = \cot x - \frac{1}{x}$

Again apply differentiation

$-\left[1/2^2 \sec^2(x/2) + \frac{1}{2^4}\sec^2\left(\frac{x}{2^2}\right) + \ldots \infty\right] = -\mathrm{cosec}^2 x + \frac{1}{x^2}$

$\therefore \frac{1}{2^2}\sec^2(x/2) + 1/2^4 \sec^2\left(\frac{x}{2^2}\right) + \ldots = \mathrm{cosec}^2 x - 1/x^2$

14. Two cyclists start from the junction of two perpendicular roads, their velocities being 3 v m/minute and 4 v m/minute. The rate at which the two cyclists are separating is

A) $\frac{7}{2}$ v m/minute **B)** 5 v m/minute **C)** v m/minute **D)** None of these

SOL:

At time t, the distance z between the cyclists is given by

$z^2 = (3vt)^2 + (4vt)^2$

$= 9v^2t^2 + 16v^2t^2$

$z^2 = 25v^2t^2$

$z = 5vt$

$\Rightarrow \frac{dz}{dt} = 5$

15. The value of $\int e^x \cdot \frac{x^2+1}{(x+1)^2} dx$ is

A) $e^x\left(\frac{x-1}{x+1}\right)+C$ **B)** $e^x\left(\frac{x+1}{x-1}\right)+C$ **C)** $e^x \cdot x + C$ **D)** None of these

SOL:

$$\int e^x \frac{(x^2+1)}{(x+1)^2}\,dx = \int e^x \cdot \frac{(x^2+2x+1-2x)}{(x+1)^2}\,dx$$

$$= \int e^x \cdot \left(1 - \frac{2x}{(x+1)^2}\right) dx$$

$$= \int e^x dx - 2\int \frac{e^x(x+1-1)}{(x+1)^2}\,dx$$

$$= \int e^x dx - 2\int \frac{e^x(x+1)}{(x+1)^2}\,dx - 2\int \frac{-e^x}{(x+1)^2}\,dx$$

$$= e^x - 2\left[\int \frac{e^x}{(x+1)}\,dx - \int \frac{e^x}{(x+1)^2}\,dx\right]$$

$$= e^x - \frac{2e^x}{1+x} + c = e^x\left(\frac{x-1}{x+1}\right) + c$$

16. The solution of the equation $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ is

A) $y = c\,(x+a)\,(1-ay)$ **B)** $y = c\,(x+a)(1+ay)$ **C)** $y = c\,(x-a)(1+ay)$ **D)** None of these

SOL:

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$y - x\frac{dy}{dx} = ay^2 + \frac{dy}{dx}a$$

$$\frac{dy}{dx}(a+x) = y - ay^2$$

Integrating

$$\Rightarrow \int \frac{dy}{y(1-ay)} = \int \frac{dx}{a+x}$$

$$\Rightarrow \int\left(\frac{1}{y} + \frac{a}{1-ay}\right) dy = \int \frac{dx}{a+x}$$

$$\Rightarrow \log y - \log(1-ay) = \log(a+x) + \log c$$

$$\log y = \log(1-ay)(a+x)$$

$$y = c(1-ay)(a+x)$$

17. The unit vector perpendicular to $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $12\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}$, is

A) $\frac{3i-3j+9k}{\sqrt{115}}$ **B)** $\frac{3i+5j-9k}{\sqrt{115}}$ **C)** $\frac{-5i+3j-9k}{\sqrt{115}}$ **D)** $\frac{5i+3j+9k}{\sqrt{115}}$

SOL:

$$a\times b = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 12 & 5 & -5 \end{vmatrix} = -5i+3j-9k$$

unit vector along $a\times b = \dfrac{-5i+3j-9k}{\sqrt{115}}$

Also $b\times a$ is $\perp$ to both a and b.

18. The angle between the straight lines whose direction consines are given by *2l + 2m – n = 0, mn + nl + lm = 0* is

A) $\frac{\pi}{2}$ **B)** $\frac{\pi}{3}$ **C)** $\frac{\pi}{4}$ **D)** none of these

SOL: $2l+2m-n=0, \quad mn+nl+lm=0$

$m(2l+2m)+l(2l+2m)+lm=0$

$\Rightarrow 2lm+2m^2+2l^2+2lm+lm=0$

$\Rightarrow 2l^2+5lm+2m^2=0$

$\Rightarrow 2l^2+4lm+lm+2m^2=0$

$\Rightarrow 2l(l+2m)+m(l+2m)=0$

$\Rightarrow (2l+m)(l+2m)=0$

$\Rightarrow l=\frac{-m}{2}$ (or) $-2m$

$l=\frac{-m}{2}$

$2\left(\frac{-m}{2}\right)+2m-n=0$

$-m+2m-n=0$

$m=n$

l	m	n
$\frac{-1}{2}$	1	1
-1	2	2

$l=-2m$

$-4m+2m-n=0$

$n=-2m$

l	m	n
-2	1	-2

$(a_1,b_1,c_1)=(-1,2,2)$

$(a_2,b_2,c_2)=(-2,1,-2)$

$\cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$= \dfrac{2+2-4}{\sqrt{9}\sqrt{9}} = 0$

$\theta = \frac{\pi}{2}$

19. What is the shortest distance of the point (1, 2, 3) from x- axis ?

A) 1 **B)** $\sqrt{6}$ **C)** $\sqrt{13}$ **D)** $\sqrt{14}$

SOL: Any point on x-axis has y = z = 0

Distance of the point (1, 2, 3) from x-axis is the distance between point (1, 2, 3) and point (1, 0, 0)

$=\sqrt{(1-1)^2+(2-0)^2+(3-0)^2}=\sqrt{2^2+3^2}$

$=\sqrt{4+9}=\sqrt{13}$

20. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14 . Then the probability that neither A nor B occurs is

A) 0.39 **B)** 0.25 **C)** 0.11 **D)** None of these

SOL: $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

=0.25+0.50-0.14=0.61

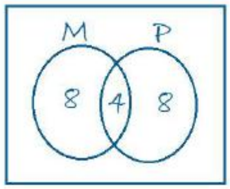
$$\therefore P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$$

=1-0.61=0.39

21. 20 teachers of a school either teach mathematics or physics, 12 of them teach mathematics, while 4 teach both the subjects. Then the number of teachers teaching physics only is

SOL:

Physics only = 8



22. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

SOL:

A signal can consist of either 2 flags, 3 flags, 4 flags or 5 flags.

There will be as many 2 flag signals as there are ways of filling in 2 vacant places in succession by the 5 flags available. By Multiplication rule, the number of ways is $5 \times 4 = 20$.

Similarly, there will be as many 3 flag signals as there are ways of filling in 3 vacant places in succession by the 5 flags.

The number of ways is $5 \times 4 \times 3 = 60$.

Continuing the same way, we find that

The number of 4 flag signals $= 5 \times 4 \times 3 \times 2 = 120$

and the number of 5 flag signals $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Therefore, the required number of signals

$= 20 + 60 + 120 + 120 = 320$.

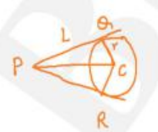
23. Two tangents PQ and PR drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ from point $P(16, 7)$. If the centre of the circle is C, then the area of quadrilateral $PQCR$ will be ___sq. units

SOL:

Area $PQCR = 2 \cdot \Delta PQC = 2 \times \frac{1}{2} L \times r$

where L = length of tangent and r = radius of circle

$L = \sqrt{S_1}$ and $r = \sqrt{1+4+20} = 5$



Hence, the required area = 75 sq.units

24. If 'P' be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot perpendicular drawn from 'P' on the directrix of the parabola, then length of each side of an equilateral triangle SMP, where 'S' is focus of the parabola is

SOL: $y^2 = 6\left(x - \frac{3}{2}\right)$

Equation of directrix is $x - \frac{3}{2} = -\frac{3}{2}$ i.e. $x = 0$

Let co-ordinates of 'P' be $\left(\frac{3}{2} + \frac{3}{2}t^2, 3t\right)$

$\therefore$ Co-ordinate of M are $(0, 3t)$

MS = $\sqrt{9 + 9t^2}$

MP = $\frac{3}{2} + \frac{3}{2}t^2$

$$9 + 9t^2 = \left[\frac{3}{2} + \frac{3}{2}t^2\right]^2 = \frac{9}{4}(1 + t^2)^2$$

$4 = 1 + t^2$

Length of side = 6

25. If $f(3) = 4$ and $f'(3) = 1$, then $\lim_{x \to 3} \frac{xf(3) - 3f(x)}{x - 3}$

SOL: We have, $\lim_{x \to 3} \frac{xf(3) - 3f(x)}{x - 3}$

$$= \lim_{x \to 3} \frac{(x-3)f(3) + 3\{f(3) - f(x)\}}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x-3)f(3)}{x - 3} + \lim_{x \to 3} \frac{-3\{f(x) - f(3)\}}{x - 3}$$

$$= f(3) + (-3)\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

$= f(3) - 3f'(3) = 4 - 3 \times 1 = 1.$

26. The value of $\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is ________

SOL:

$$I = \int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx \quad — ①$$

$$= \int_0^{\pi/2} \log\left[\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)}\right] dx$$

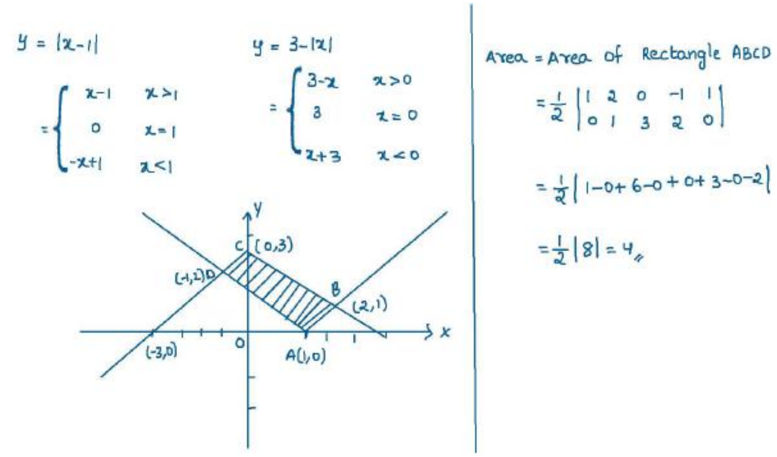
$$= \int_0^{\pi/2} \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) dx \quad — ②$$

$$① + ② \Rightarrow 2I = \int_0^{\pi/2} 0 \, dx$$

$$\Rightarrow I = 0$$

27. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is ___sq units

SOL:



28. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then β =___

SOL: If $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c} = p\vec{a} + q\vec{b}$

i.e $\hat{i} + \alpha\hat{j} + \beta\hat{k} = p(\hat{i} + \hat{j} + \hat{k}) + q(4\hat{i} + 3\hat{j} + \hat{k})$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ we get

1 = p + 4q, α = p + 3q, β = p + 4q

From first and third, β = 1

Now, $|\vec{c}| = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow 1 + \alpha^2 + 1 = 3 \Rightarrow \alpha = \pm 1$

Hence, α = ±1, β=1

29. If the angle between the planes 2x − y + 2z = 3 and 3x + 6y + cz = 4 is $\cos^{-1}\left(\frac{4}{21}\right)$, then c^2 =

SOL: d.r.'s of the normals are (2, −1, 2) and (3, 6, c)

According to question, $\theta = \cos^{-1}\left(\frac{4}{21}\right)$

$\Rightarrow \frac{6-6+2c}{3\sqrt{45+c^2}} = \frac{4}{21} \Rightarrow c^2 = 4$

30. A problem in mathematics is given to three students A, B and C and their respective probability of solving the problem is $1/2, 1/3$ and $1/4$ then find the probability that the problem is solved.

SOL: $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{3}{4}$

Physics

31. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does the body take to stop?

A) 2s **B)** 4s **C)** 6s **D)** 8s

SOL:

Given, $v = 15\ m/sec$, $m = 20\ kg$, $F = 50$

$\frac{1}{2}mv^2 = F.S$

$\frac{1}{2} \cdot 20 \times (15)^2 = 50 \times S$

$S = 45\ m$

$v^2 - u^2 = 2aS$

$v = u + at$

$0 = 15 + at \Rightarrow t = \frac{-15}{a}$

$S = ut + \frac{1}{2}at^2$

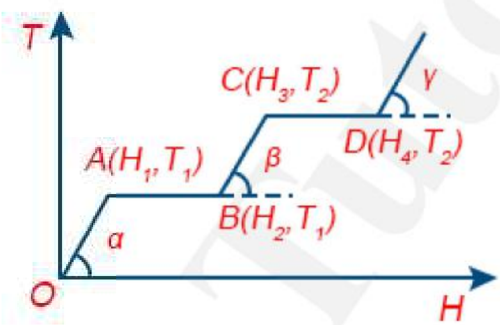
$45 = 15 \times t + \frac{1}{2}at^2$

$45 = 15t - \frac{7.5}{t} \cdot t^2$

$7.5t = 45$

$t = 6\ sec$

32. The graph shows the variation of temperature (T) of one kilogram of a material with the heat (H) supplied to it. At O, the substance is in the solid state. From the graph, we can conclude that



A) T_2 is the melting point of the solid **B)** BC represents the change of state from solid to liquid

C) $(H_2 - H_1)$ represents the latent heat of vaporization of the liquid

D) $(H_3 - H_1)$ represents the latent heat of vaporization of the liquid

SOL:

As we observe the figure.

In the region AB temperature is constant therefore at this temperature phase of the material changes from solid to liquid and $(H_2 - H_1)$ heat will be absorb by the material. This heat is known as the heat of melting of the solid.

Similarly, in the region CD temperature is constant. Therefore at this temperature phase of the material changes from liquid to gas and $(H_4 - H_3)$ heat will be absorbed by the material. This heat is known as heat of vapourisation of liquid.

So option 'C' $(H_2 - H_1)$ represents the latent heat of fusion of the substance is correct option.

33. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

A) $C_p - C_v = \frac{R}{28}$ **B)** $C_p - C_v = \frac{R}{14}$ **C)** $C_p - C_v = R$ **D)** $C_p - C_v = 28R$

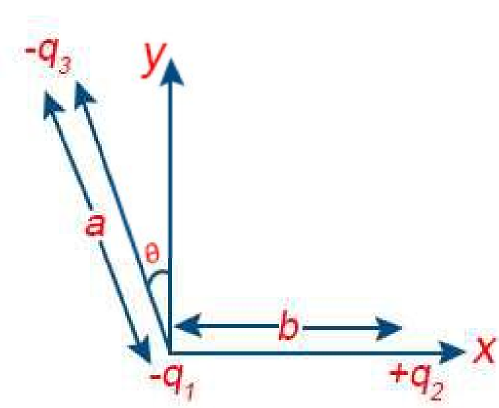
SOL:

$C_p - C_v = r$

but $r = \frac{R}{M}$

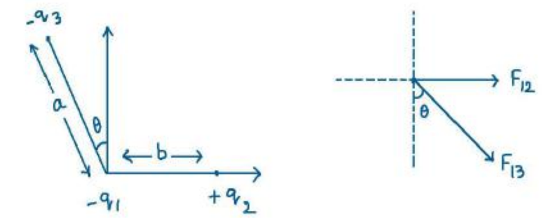
$C_p - C_v = \frac{R}{28}$

34. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



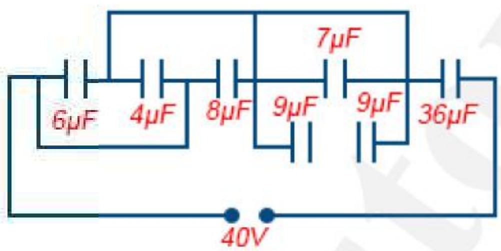
A) $\frac{q_2}{b^2} - \frac{q_3}{a^2} cos\theta$ **B)** $\frac{q_2}{b^2} + \frac{q_3}{a^2} sin\theta$ **C)** $\frac{q_2}{b^2} + \frac{q_3}{a^2} cos\theta$ **D)** $\frac{q_2}{b^2} - \frac{q_3}{a^2} sin\theta$

SOL:



$$F_x = F_{13}\sin\theta + F_{12}$$
$$= \frac{q_1 q_3}{a^2}\sin\theta + \frac{q_1 q_2}{b^2}$$
$$= q_1\left[\frac{q_3\sin\theta}{a^2} + \frac{q_2}{b^2}\right]$$

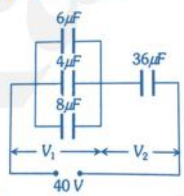
35. In the following diagram, the charge and potential difference across 8 μF capacitance will be respectively



A) 320 μC, 40 V **B)** 420 μC, 50 V **C)** 214 μC, 27 V **D)** 360 μC, 45 V

SOL:

Given circuit can be redrawn as follows capacitors $9\mu F$, $9\mu F$ and $7\mu F$ are short circuited. So they are deleted



$V_1 + V_2 = 40V$ — ①

$\frac{V_1}{V_2} = \frac{36}{18} = 2$

$V_1 = 2V_2$ — ②

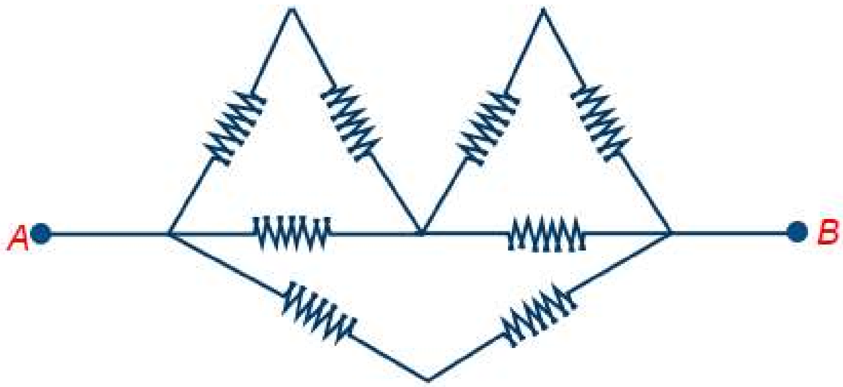
from ①, $2V_2 + V_2 = 40V \Rightarrow 3V_2 = 40V \Rightarrow V_2 = \frac{40}{3}V$

Then $V_1 = \frac{40}{3}V = 40V \Rightarrow V_1 = \frac{80}{3}V$.

Charge on $8\mu F$ capacitors $= 8 \times \frac{80}{3} = 213.3\mu F \approx 214\mu F$

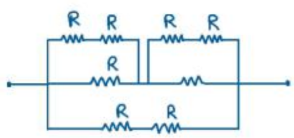
we have potential difference, $V = \frac{Q}{C} \Rightarrow V = \frac{214}{8} \approx 27V$

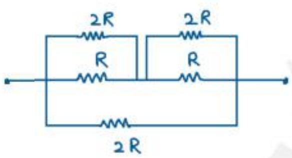
36. What is the equivalent resistance between A and B? (Each resistor has resistance R)



A) $\frac{4R}{3}$ **B)** $\frac{5R}{3}$ **C)** $\frac{4R}{5}$ **D)** $\frac{3R}{4}$

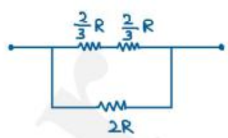
SOL:



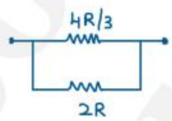


$\frac{1}{R_e} = \frac{1}{2R} + \frac{1}{R}$

$\frac{1}{R_e} = \frac{R+2R}{2R^2} \Rightarrow \frac{2}{3}R = R_e$



$R_e = \frac{2}{3}R + \frac{2}{3}R = \frac{4R}{3}$



$\frac{1}{R_e} = \frac{3}{4R} + \frac{1}{2R} = \frac{6R+4R}{8R^2}$

$= \frac{6R+4R}{8R^2} = \frac{10R}{8R^2} = \frac{4R}{5}$

37. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be

A) 2 s **B)** $\frac{2}{3}s$ **C)** $2\sqrt{3}s$ **D)** $\frac{2}{\sqrt{3}}s$

SOL: We know $T = 2\pi\sqrt{\frac{I}{MH}}$ where $I \rightarrow$ moment of inertia of magnet

$= \frac{mL^2}{12}$ (or) m = pole strength $\times L$

$I' = \frac{1}{12}\left[\frac{m}{3}\right]\left[\frac{L}{3}\right]^2 \times 3 = \frac{ML^2}{108}$

When 3 equal parts of magnets are placed on one another with their like poles together.

$\Rightarrow M' =$ pole strength $\times \frac{L}{3} \times 3 = M$

So $T' = 2\pi\sqrt{\left(\frac{I/9}{M+I}\right)} \Rightarrow T' = \frac{1}{3} \times T$

$\Rightarrow T' = \frac{1}{3} \times 2 = \frac{2}{3}$

38. What will be de-Broglie wavelength of an electron having kinetic energy of 500 eV. Given h = 6.6×10^{-34} JS, e = 1.6×10^{-19} C, $m_e = 9.11 \times 10^{-31}$ kg.

A) 0.5467 Å **B)** 0.5222 Å **C)** 1.5267 Å **D)** 2.555 Å

SOL:

Here , $\lambda = ?$

$h = 6.6 \times 10^{-34}$ JS, $e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg

K.E of electron , $\frac{1}{2} m v^2 = 500$ eV

$= 500 \times 1.6 \times 10^{-19}$ J

$m v^2 = 2 \times 500 \times 1.6 \times 10^{-19}$ J

$m^2 v^2 = 9.11 \times 10^{-31} \times 2 \times 500 \times 1.6 \times 10^{-19}$

$m v = \sqrt{9.11 \times 1.6 \times 10^{-47}}$

$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{\sqrt{9.11 \times 1.6 \times 10^{-47}}}$$

$= 0.5467 \times 10^{-10}$ m $= 0.5467$ Å

39. Calculate the impact parameter of a 5 MeV alpha particle scattered by 10° when it approaches a gold nucleus. Take Z = 79 for gold.

A) 2.6×10^{-13} m **B)** 3.6×10^{-1} m **C)** 4.6×10^{-12} m **D)** 5.6×6^{-1} m

SOL:

Here K.E $= \frac{1}{2} m v^2 = 5$ MeV

$= 5 \times 1.6 \times 10^{-13}$ J

$\theta = 10^\circ$, $Z = 79$, $b = ?$

$$\text{As } b = \frac{Ze^2 \cot \theta/2}{4\pi\varepsilon_0 \left(\frac{1}{2} m v^2\right)}$$

$$b = \frac{9 \times 10^9 \times 79 (1.6 \times 10^{-19})^2 \cot 5^\circ}{5 \times 1.6 \times 10^{-13}}$$

$$= \frac{9 \times 79 \times 1.6 \times 1.6 \times 10^{-16}}{8 \times 0.0875} \quad (\tan 5^\circ = 0.0875)$$

$b = 2.6 \times 10^{-13}$ m

40. The nuclear radius of ${}_8O^{16}$ is 3×10^{-15} metre. If an atomic mass unit is 1.67×10^{-27} kg, then the nuclear density is approximately :

A) 2.35×10^{17} gm per cm^3 **B)** 2.35×10^{17} kg per metre^3 **C)** 2.35×10^{17} gm per metre^3

D) 2.35×10^{17} kg per cm^3

SOL:

For nucleus of ${}_{8}O^{16}$:

Mass $= (16)(1.67 \times 10^{-27})$ kg

Volume $= \frac{4}{3}\pi R^3$

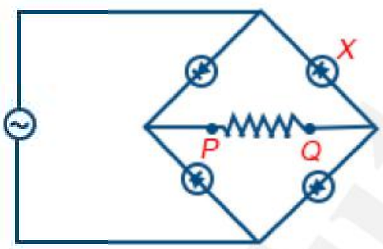
$= \frac{4}{3}\pi (3 \times 10^{-15})^3$ metre^3

$= 36\pi \times 10^{-45}$ metre^3

Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{16 \times 1.67 \times 10^{-27} \text{ kg}}{36\pi \times 10^{-45} \text{ metre}^3}$

$= 2.35 \times 10^{17}$ kg/metre^3

41. The figure shows a bridge rectifier with a sinusoidal alternating voltage applied to it, the output terminals *P* and *Q* being joined together by a load resistance. If the diode *X* were removed leaving a break in the circuit, which trace would be seen on a cathode-ray oscilloscope connected across *PQ*?



A)  **B)**  **C)**  **D)**

SOL:

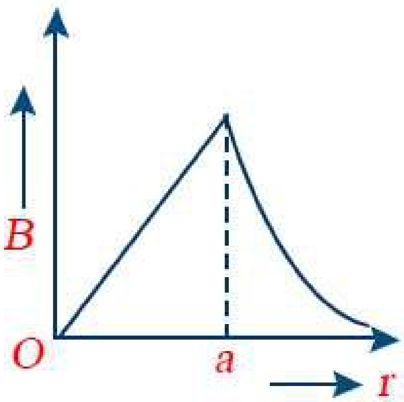
When X is removed

For positive cycles - diode is forward biased

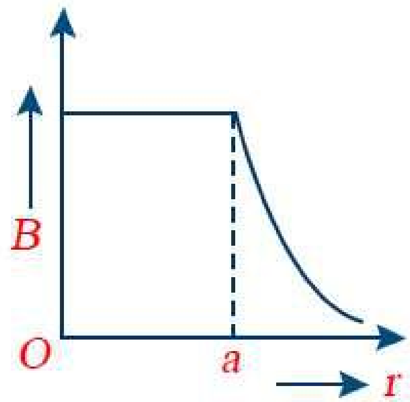
For negative cycles - diode is negative biased

Hence output is obtained only positive cycles.

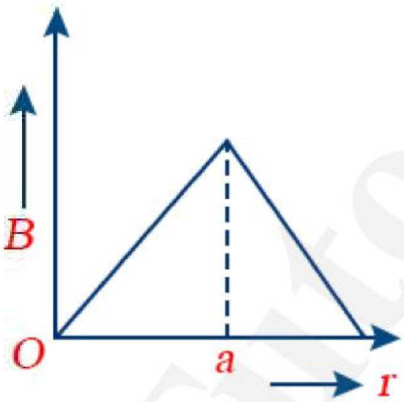
42. The magnetic field due to a straight conductor of uniform cross-section of radius a and carrying a steady current is represented by



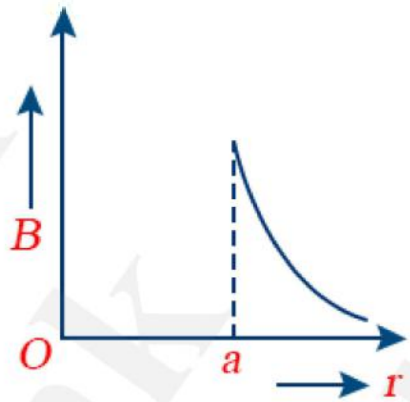
A)



B)



C)

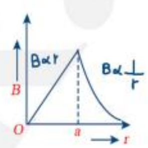


D)

SOL:

The magnetic field at a point outside the straight conductor is given by $B = \frac{\mu_0 I}{2\pi r}$

It means $B \propto \frac{1}{r}$ (if $r > a$)



The magnetic field at a point inside the conductor is $B = \frac{\mu_0 I \times r}{2\pi a^2}$

$B \propto r$ (if $r < a$)

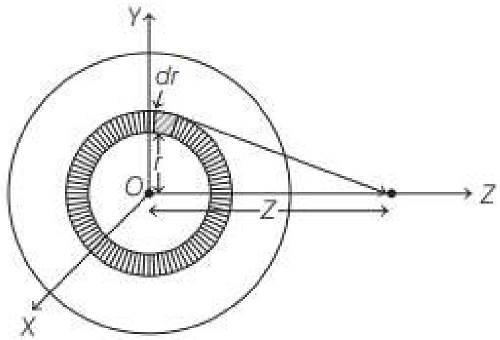
So option 'a' is correct.

43. A uniformly charged disc of radius R having surface charge density σ is placed in the xy-plane with its centre at the origin. Find the electric field intensity along the Z-axis at a distance Z from origin

A) $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{\sqrt{(Z^2+R^2)}}\right)$ **B)** $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{\sqrt{(Z^2+R^2)}}\right)$ **C)** $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{\sqrt{(Z^2+R^2)}} + Z\right)$

D) $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{\sqrt{(Z^2+R^2)}} + \frac{1}{Z^2}\right)$

SOL: A circular disc is placed in xy-plane with its centre at the origin as shown below



Consider an elemental ring of thickness dr and radius r. Now, the area of the elemental ring can be given by

$$dA = 2\pi r dr$$

The charge stored in this elemental ring,

$$dq = \sigma dA$$

Now, the electric field at the point on Z-axis at a distance of Z from origin can be given by

$$dE = \frac{kdqZ}{\left(r^2+Z^2\right)^{\frac{3}{2}}}$$

Substituting the value of dq and dA in above equation, we get

$$dE = \frac{kZ\sigma(2\pi r dr)}{\left(r^2+Z^2\right)^{3/2}}$$

$$= \frac{\sigma Z}{2\varepsilon_0}\left(\frac{rdr}{\left(r^2+Z^2\right)^{3/2}}\right)$$

Calculating the total electric field by integrating the above expression from $r = 0$ to $r = R$, we get

$$E = \frac{\sigma Z}{2\varepsilon_0}\int_0^R \frac{rdr}{\left(r^2+Z^2\right)^{3/2}}$$

Put $r^2 + z^2 = u^2$

$$\Rightarrow 2rdr = 2udu$$

$$\Rightarrow rdr = udu$$

For lower limit, $r = 0 \Rightarrow u = Z$

Upper limit, $r = R \Rightarrow u = \sqrt{R^2 + Z^2}$

$$E = \frac{\sigma Z}{2\varepsilon_0}\int_Z^{\sqrt{R^2+Z^2}} \frac{udu}{u^3}$$

$$E = \frac{\sigma Z}{2\varepsilon_0}\int_Z^{\sqrt{R^2+Z^2}} \frac{du}{u^2}$$

$$= \frac{\sigma Z}{2\varepsilon_0} \int_Z^{\sqrt{R^2+Z^2}} u^{-2} du$$

$$= \frac{\sigma Z}{2\varepsilon_0} \left[\frac{u^{-2+1}}{(-2+1)}\right]_Z^{\sqrt{Z^2+R^2}}$$

$$= \frac{\sigma Z}{2\varepsilon_0} \left[-\frac{1}{u}\right]_Z^{\sqrt{Z^2+R^2}}$$

$$= \frac{\sigma Z}{2\varepsilon_0} \left[-\frac{1}{\sqrt{(Z^2+R^2)}} + \frac{1}{Z}\right]$$

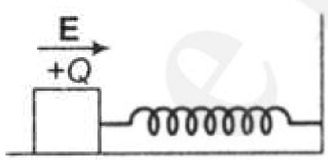
$$= \frac{\sigma Z}{2\varepsilon_0} \left[\frac{1}{Z} - \frac{1}{\sqrt{(Z^2+R^2)}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{Z}{\sqrt{(Z^2+R^2)}}\right]$$

Thus, the electric field at the point on Z-axis at a distance of Z from origin is

$\frac{\sigma}{2\varepsilon_0} \left[1 - \frac{Z}{\sqrt{(Z^2+R^2)}}\right]$.

44. A wooden block performs SHM on a frictionless surface with frequency v_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field E is switched on as shown, then the SHM of the block will be



A) Of the same frequency and with shifted mean position

B) Of the same frequency and with the same mean position

C) Of changed frequency and with shifted mean position

D) Of changed frequency and with the same mean position

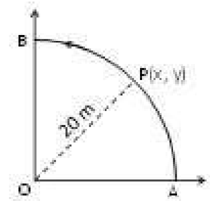
SOL: Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position.

For example, in the given equation mean position is at natural length of spring in the absence of electric field. Whereas in the presence of electric field mean position will be obtained after a compression of x_0.

Where x_0 is given by
$\mathbf{kx_0 = QE}$

$x_0 = \frac{QE}{k}$

45. A point P moves in counterclockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is $20\ \mathrm{m}$. The



acceleration of P when $t = 2\ \mathrm{s}$ is nearly

A) $13\ \mathrm{ms}^{-2}$ **B)** $12\ \mathrm{ms}^{-2}$ **C)** $7.2\ \mathrm{ms}^{-2}$ **D)** $14\ \mathrm{ms}^{-2}$

SOL: Required Solution

46. A SHM is represented by $x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of the SHM is

A) 10 cm **B)** 20 cm **C)** $5\sqrt{2} cm$ **D)** 50 cm

SOL: Required Solution

47. The amplitude of a mechanical wave along the positive x - direction is $y = \frac{1}{(1+x^2)}$ at t = 0 ; and $y = \frac{1}{\left[1+(x-2)^2\right]}$ at t = 8s

Where x and y are in meters. The shape of the wave is constant during its propagation. What is the velocity of the wave?

A) 0.5 m / s **B)** 1.0 m / s **C)** 1.5 m / s **D)** 2.0 m / s

SOL: Required Solution

48. The mean time period of second's pendulum is 2.00 s and mean absolute error in the time period is 0.05 s. To express maximum estimate of error, the time period should be written as

A) $(2.00 \pm 0.01)\mathrm{s}$ **B)** $(2.00 + 0.025)\mathrm{s}$ **C)** $(2.00 \pm 0.05)\mathrm{s}$ **D)** $(2.00 \pm 0.10)\mathrm{s}$

SOL: Given,

Mean value of time period $T_0 = 2.00\mathrm{sec}$

Mean absolute error in time period $(\overline{\Delta T}) = 0.05\mathrm{sec}$

To express maximum estimate of error, the error in time period should be written as,

$(2.00 \pm 0.05)\mathrm{sec}$

49. $y = (\sin x + \cos x)^x$, then $\frac{dy}{dx}$ is

A) $y(\sin x + \cos x)$ **B)** $y\left\{\frac{\log y}{x} + \frac{x(\cos x - \sin x)}{\sin x + \cos x}\right\}$ **C)** $y\left(\frac{\log y}{x} + y^2\right)$ **D)** none of these

SOL: To find the derivative $\frac{dy}{dz}$ of the function y – (sin x + cos x)^X with respect to x, we can use the chain rule and the power rule.

Given that $y – (\sin x + \cos x)^{x}$, we need to differentiate y with respect to x.

Let's denote the inner function $u – \sin x + \cos x$. Then $y – u^{z}$.

Using the chain rule, the derivative of y with respect to x is :

$$\frac{dy}{dx} - \frac{dy}{du}, \frac{du}{dz}$$

First, let's find $\frac{dy}{du}$:

$$\frac{dy}{du} - x.\, u^{x-1}$$

Now, let's find $\frac{du}{dx}$:

$$\frac{d}{dx}(\sin x + \cos x) - \cos x - \sin x$$

Now, putting it all together:

$$\frac{dy}{dx} - \frac{dy}{du}, \frac{du}{dz} - x.\ (\sin x + \cos x)^{x-1}, (\cos x - \sin x)$$

So, the derivative $\frac{dy}{dx}$ is :

$$\frac{dy}{dx} - x(\sin x + \cos x)^{x-1}(\cos x - \sin x)$$

50. Which of the following activities involve the utmost expression of passion, talent and intelligence?

A) literature **B)** science **C)** music **D)** all the above

SOL: all the above

51. How many hours would make a day if the Earth were rotating at such a high speed that the weight of a body on the equator were zero________.

SOL:

$$g\phi = g - R\omega^2 \cos^2\phi$$

$$0 = g - R\omega^2$$

$$\omega = \sqrt{g/R}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{R}}$$

$$\frac{2 \times 3.14}{T} = \sqrt{\frac{9.8}{6400 \times 10^3}}$$

$$T = 1.4\ hr$$

52. When a weight of 5 kg is suspended from a copper wire of length 30 m and diameter 0.5 mm, the length of the wire increases by 2.4 cm. If the diameter is doubled, the extension produced is(in cm)_________.

SOL:

$$\frac{e_1}{e_2} = \frac{F_1}{F_2} \times \frac{L_1}{L_2} \times \frac{A_2}{A_1} \times \frac{Y_2}{Y_1}$$

$$\frac{e_1}{e_2} = \frac{30}{30.0} \times \left(\frac{2R}{R}\right)^2$$

$$\frac{2.4}{e_2} = \frac{4}{1}$$

$$e_2 = 0.6$$

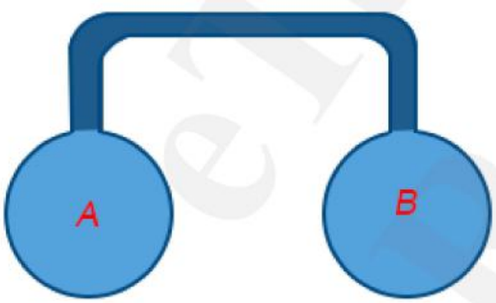
53. 5 moles of an ideal diatomic gas ($\gamma = 1.4$) are heated at a constant pressure. If 280 J of heat energy is supplied to the gas the work done by the gas is _____ J

SOL: $\Delta Q = nC_p \Delta T = 280$ J.

$\Delta U = nC_v\, \Delta T = \mathbf{n} \times \frac{C_p}{\gamma} \mathbf{\Delta T = 200}$ **J**

$\Delta W = \Delta Q - \Delta U = 280 - 200 = 80$ J

54. Two spherical vessel of equal volume, are connected by a narrow tube. The apparatus contains an ideal gas at one atmosphere and $300K$. Now if one vessel is immersed in a bath of constant temperature $600K$ and the other in a bath of constant temperature $300K$. Then the common pressure in atm will be______.



SOL: Initially both are at same temp, and are at same pressure
when they are inserted in bath

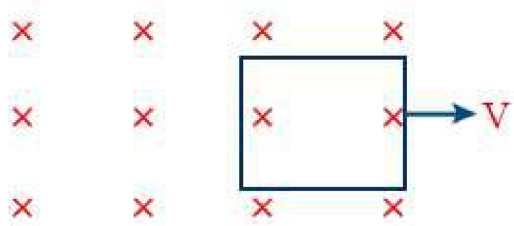
$T_1 = 300\,k \quad T_2 = 600\,k$

$P_1 = 1\,atm \quad P_2 = 2\,atm$

$$P = \frac{P_1 T_2 + P_2 T_1}{T_1 + T_2}$$

$$P = \frac{4}{3}\ atm$$

55. Figure shows a square loop of side 0.5m and resistance 10Ω. The magnetic field has a magnitude B = 1.0 T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 second is $n \times 10^{-3}$ J.Then the value of 'n' is ___



SOL: Speed of the loop $v = \frac{l}{t} = \frac{0.5}{2} = 0.25 m/s$

Induced emf = e = Bvl = (1.0) × (0.25) × (0.5)

= 0.125 V

Current in the loop $i = \frac{e}{R} = \frac{0.125}{10} = 1.25 \times 10^{-2} A$

Magnetic force on the loop = $F_m = IlB = \left(1.25 \times 10^{-2} \times \frac{1}{2} \times 1\right)$

$= 0.625 \times 10^{-2}$

$= 6.25 \times 10^{-3}$ N

workdone w = F × S

$= 6.25 \times 10^{-3} \times \frac{1}{2}$

$= 3.125 \times 10^{-3}$ J

56. In a young's double slit arrangement the distance between the slits is by illuminated by monochromatic light of wavelength λ = 6000°A is 1 mm and the distance of the screen from the slits is 60 cm. The least distance of a point on a screen from the central maxima where the intensity is 3/4th the of the maximum intensity is______μm.

SOL: $I = I_0 \cos^2 \delta/2$

$3/4\ I_0 = I_0 \cos^2 \delta/2$

Path difference $\Delta x = \frac{\lambda}{6}$

$x = \frac{\Delta x \times D}{d} = \frac{\lambda}{6} \times \frac{D}{d}$

$x = \frac{6000 \times 10^{-10}}{6} \times \frac{60 \times 10^{-2}}{1 \times 10^{-3}}$

$= 1000 \times 10^{-10} \times 6 \times 10^{2}$

$= 6000 \times 10^{-8}$

$= 60 \times 10^{-6}$ m

x = 60 μm

57. Which state of triply ionised beryllium (Be^{++}) has the same orbital radius as that of the ground state of hydrogen at n is______

SOL:

Given : Energy level of ground state of hydrogen $(n_1) = 1$

We know that atomic number of hydrogen atom $(z_1) = 1$

and atomic number of beryllium atom $(z_2) = 4$

We also know that orbital radius,

$$r = 4\pi\varepsilon_0 \times \frac{n^2 h^2}{4\pi me^2 z} \quad \text{or} \quad n \propto \sqrt{z}$$

Since, orbital radius of ionised beryllium is equal to the orbital radius of hydrogen atom in ground state, therefore

$$\frac{n_1}{n_2} = \sqrt{\frac{z_1}{z_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{or } n_2 = 2n_1 = 2 \times 1 = 2$$

58. The radius of germanium (Ge) nuclide is measured to be twice the radius of ${}_4Be^9$. The number of nucleons in Ge are_____

SOL:

Nuclear radius. $R = R_0 (A)^{1/3}$ (where $R_0 = 1.2$ fermi)

$$\text{or } R \propto (A)^{1/3}$$

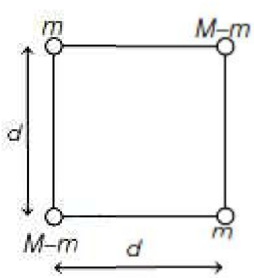
$$\therefore \frac{R_{Be}}{R_{Ge}} = \frac{(9)^{1/3}}{(A)^{1/3}} \quad \text{or} \quad \frac{R_{Be}}{2R_{Ge}} = \frac{(9)^{1/3}}{(A)^{1/3}}$$

$$\therefore (A)^{1/3} = 2 \times (9)^{1/3}$$

$$\text{or } A = 2^3 \times 9 = 72$$

The number of nucleons in Ge is 72

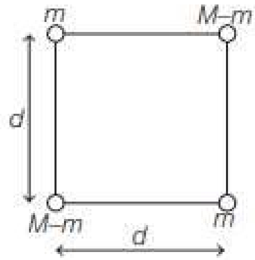
59. A body of mass $2M$ splits into four masses $\{m, M-m, m, M-m\}$, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is $x : 1$. The value of x is......... .



SOL: Given, total mass of body is $2M$.

Potential energy is maximum at $M/m = x/1$

The arrangement of masses to form a square is shown in diagram.



The gravitational potential energy of a body is given

$U=-\frac{GMm}{r}$

For the given system, the potential energy will be

$U_T=-\frac{Gm(M-m)}{d}-\frac{Gm(M-m)}{d}-\frac{Gm(M-m)}{d}-\frac{Gm(M-m)}{d}-\frac{Gm^2}{(\sqrt{2}d)^2}-\frac{G(M-m)^2}{(\sqrt{2}d)^2}$

$U_T=-\frac{4Gm(M-m)}{d}-\frac{Gm^2}{\sqrt{2}d}-\frac{G(M-m)^2}{\sqrt{2}d}$

For maximum potential energy,

$\frac{dU_T}{dm}=0$

$-\frac{4G}{d}[M-2m]-\frac{G}{\sqrt{2}d}[2m]$

$-\frac{G}{\sqrt{2}d}[2(M-m)\times-1]=0$

$\Rightarrow 4M-8m+\sqrt{2}m=\sqrt{2}(M-m)$

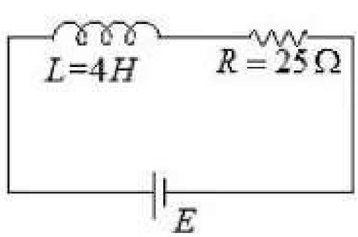
$(4-\sqrt{2})M=(8-2\sqrt{2})m$

$\frac{M}{m}=\frac{2(4-\sqrt{2})}{4-\sqrt{2}}=2$

Comparing it with the given value, we get $x=2$

Thus, potential energy will be maximum when $x=2$

60. In the given figure, an inductor and resistor are connected in series with a battery of emf E volt. $\frac{E^a}{2b}\ \mathrm{J/s}$ represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be _________.



SOL: $i=\frac{E}{R}\left[1-e^{-Rt/L}\right]$

$$E_{\text{induction}} = \frac{1}{2}Li^2 = \frac{1}{2}L\frac{E^2}{R^2}\left[1-e^{-Rt/L}\right]^2 = x \text{ (let)}$$

$$\Rightarrow \frac{dx}{dt} = 2\left[1-e^{-Rt/L}\right]\cdot e^{-Rt/L}\cdot\frac{R}{L}\cdot\frac{E^2 L}{2R^2}$$

$$= \frac{E^2}{R}\left[e^{-Rt/L}-e^{-2Rt/L}\right]$$

For maximum rate, $\frac{d^2x}{dt^2} = 0$

$$\Rightarrow e^{-Rt/L} = 2e^{-2Rt/L} \Rightarrow e^{Rt/L} = 2$$

$$\Rightarrow \left(\frac{dx}{dt}\right)_{\text{max.}} = \frac{E^2}{R}\left[2^{-1}-2^{-2}\right] = \frac{E^2}{4R}$$

⇒ a = 2 and b = 2R = 50

$$\Rightarrow \frac{b}{a} = 25$$

Chemistry

61. The average molecular mass of a mixture of gas containing nitrogen and carbondioxide is 36. The mixture contain 140 g of nitrogen, therefore the amount of CO_2 present in the mixture is

A) 213 g **B)** 200 g **C)** 313 g **D)** 220 g

SOL: $36 = \frac{140 + x \times 44}{(5+x)}$

36 (5 + x) = 140 + 44 x

180 + 36 x = 140 + 44 x

40 = 8 x

x = 5 moles

∴ Mass of CO_2 = 5 × 44 = 220 gms

62. The first ionization potential of Na, Mg, Al and Si are in the order:

A) Na > Mg > Al < Si **B)** Na > Mg > Al > Si **C)** Na < Mg < Al < Si **D)** Na < Mg > Al > Si

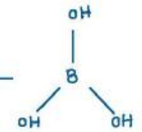
SOL: The first ionization potential of Na, Mg, Al and Si are in the order Na < Mg > Al > Si, due to completely filled sub shell electronic configuration i.e., $3s^2$

63. The states of hybridisation of boron and oxygen atoms in boric acid (H_3BO_3) are respectively

A) sp^2 and sp^2 **B)** sp^2 and sp^3 **C)** sp^3 and sp^2 **D)** sp^3 and sp^3

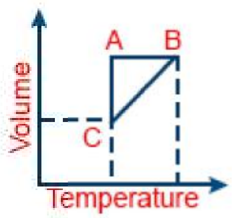
SOL:

Boric acid (H_3BO_3) ——



Boron undergoes sp^2 hybridisation. oxygen has sp^3 hybridisation.

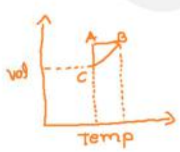
64. Five moles of a gas is put through a series of changes as shown graphically in a cyclic process the A → B, B → C and C → A respectively are



A) Isochoric, Isobaric, Isothermal **B)** Isobaric, Isochoric, Isothermal **C)** Isothermal, Isobaric, Isochoric

D) Isochoric, Isothermal, Isobaric

SOL:



Volume remained constant from A → B, so isochoric
pressure remained constant from B → C, so isobaric
Temperature remained constant from C → A, so isothermal

65. In the reaction,
$I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$, equivalent weight of iodine will be equal to

A) twice its molecular weight **B)** its molecular weight **C)** ½ its molecular weight **D)** ¼ its molecular weight

SOL:

$$\overset{0}{I_2} + 2S_2O_3^{2-} \longrightarrow 2I^- + S_4O_6^{2-}$$

so, the equivalent weight

$\Rightarrow \frac{1}{2}$ [its molecular weight]

66. When 0.25 g of an organic compounds is heated with HNO_3 and $AgNO_3$ in a carius tube, it gives 0.35 g of silver chloride. The percentage of chlorine in the compound is

A) 36.6% **B)** 45.3% **C)** 34.6% **D)** 54.8%

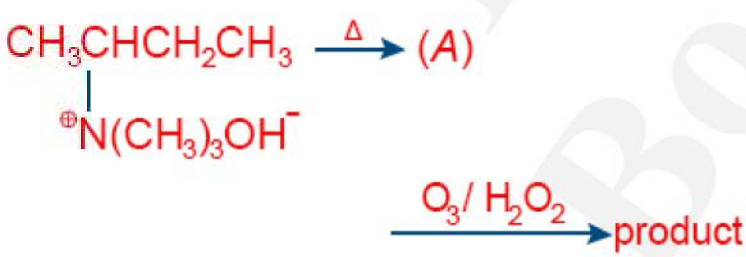
SOL:

% cl in the compound

$$= \frac{35.5 \times 0.35 \times 100}{143.5 \times 0.25}$$

$$= 34.6\%$$

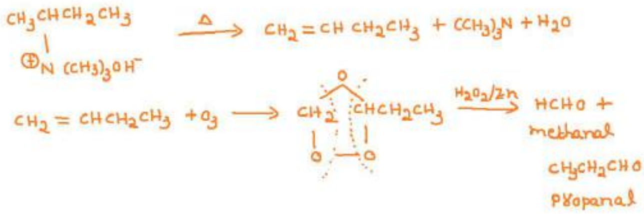
67. A major alkene (*A*) obtained in the following reaction undergo ozonolysis to give the product.



The product obtained is/are identified as

A) ethanal only **B)** methanal and propanal **C)** methanal and acetone **D)** Only acetone

SOL:



68. An aqueous solution of sugar undergoes acid catalysed hydrolysis. 50 g sugar in 125 mL water rotates the plane of plane polarized light by + 13.1° at t = 0. After complete hydrolysis, it shows a rotation of −3.75°. The percentage hydrolysis of sugar at time 't' in the same solution having a rotation of 5° is

A) 42 % **B)** 58 % **C)** 48 % **D)** 55 %

SOL:

Let 'a' be the initial concentration.
then the final concentration is a−x.

Now,
degree of hydrolysis of sugar $= \frac{x}{a}$

$$= \frac{R_t - R_0}{R_\infty - R_0}$$

$$= \frac{5-13.1}{-37.5-13.1}$$

$$= \frac{-8.1}{-16.85}$$

$$= 0.48$$

∴ percentage of hydrolysis of sugar = 0.48 × 100
= 48 %

69. The complex compound used in the chemotheraphy of cancer is

A) cis-$[Pt^{IV}(NH_3)_2Cl_4]$ **B)** trans-$[Pt^{II}(NH_3)_2Cl_2]$ **C)** cis-$[Pt^{IV}(NH_3)_4Cl_2]Cl_2$ **D)** cis-$[Pt^{II}(NH_3)_2Cl_2]$

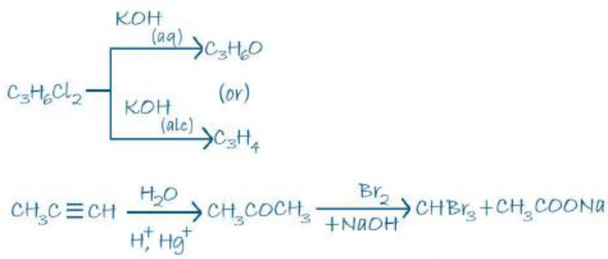
SOL:

In chemotherapy of cancer, the complex compound used is cis − $[Pt^{II}(NH_3)_2Cl_2]$.

70. A compound A of formula $C_3H_6Cl_2$ on reaction with alkali can give B of formula C_3H_6O or C of formula C_3H_4. B on oxidation gives a compound of the formula $C_3H_6O_2$. C with dilute H_2SO_4 containing Hg^{2+} ion gives D of formula C_3H_6O, which with bromine and NaOH gives the sodium salt of $C_2H_4O_2$. Then A is

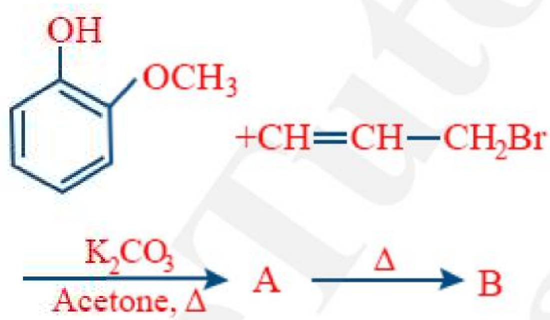
A) $CH_3CH_2CHCl_2$ **B)** $CH_2CCl_2CH_3$ **C)** $CH_2ClCH_2CH_2Cl$ **D)** $CH_3CHClCH_2Cl$

SOL:

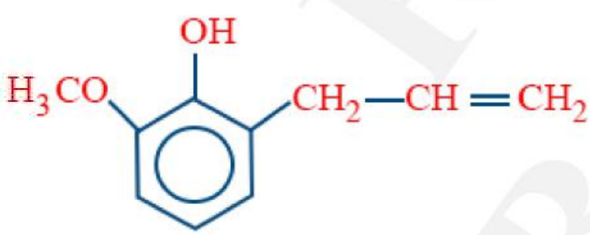


Since, B and D are different, then B is CH_3CH_2CHO and so A is $CH_3CH_2CHCl_2$.

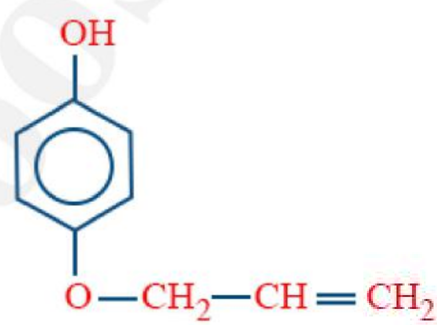
71.



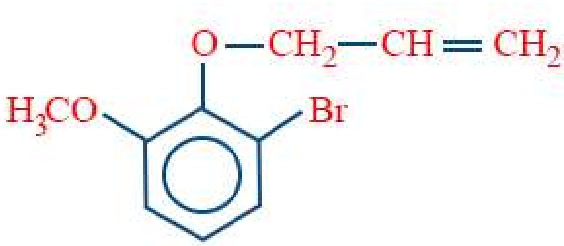
The structure of product B is given as



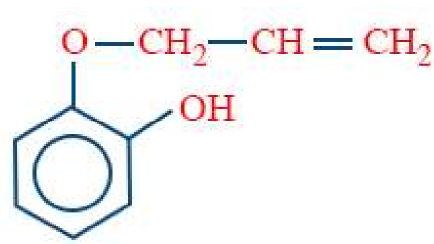
A)



B)

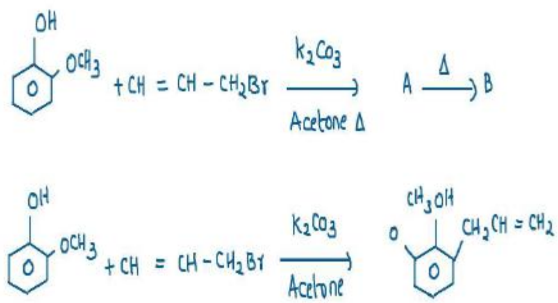


C)

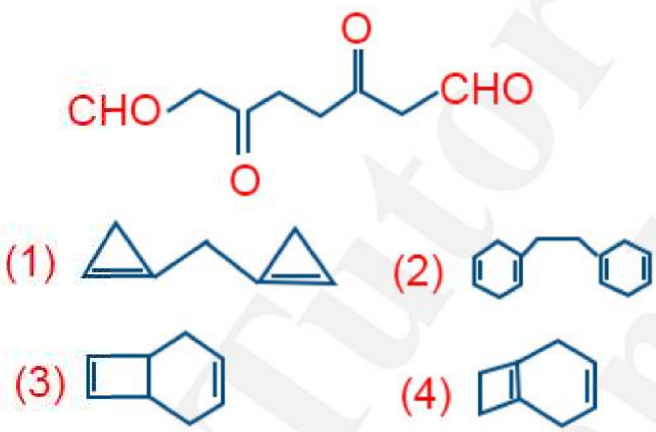


D)

SOL:

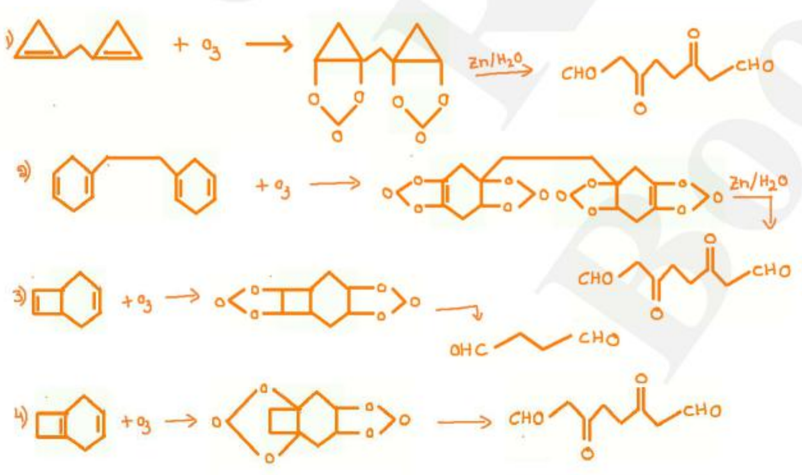


72. Which of the following compounds will give this product on ozonolysis?

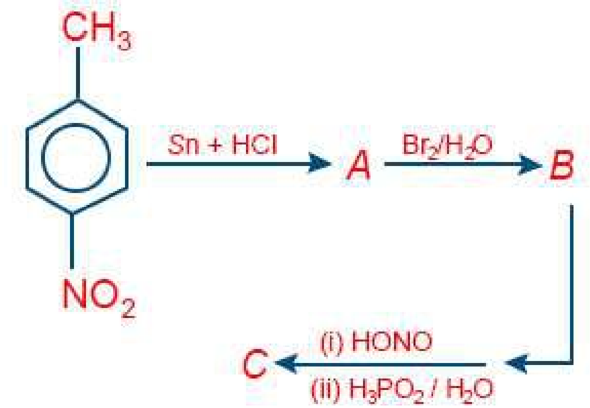


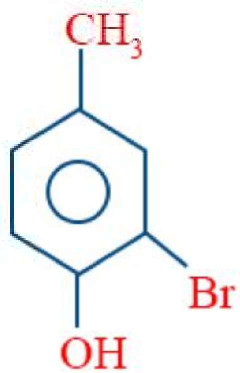
A) I, II IV **B)** I, II, III **C)** I, II **D)** II, IV

SOL:

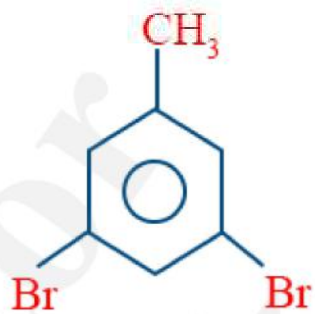


73. The final product *C* obtained in the following sequence of reactions

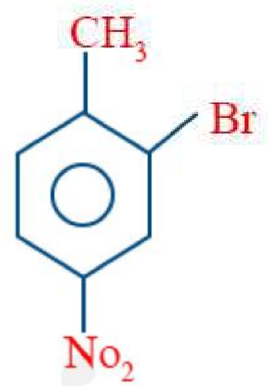

CH3
NO2
Sn + HCl
A
Br2/H2O
B
C
(i) HONO
(ii) H3PO2 / H2O


CH3
Br
OH

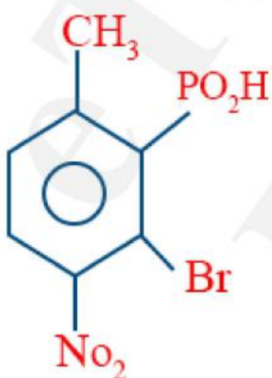
A)


CH3
Br
Br

B)

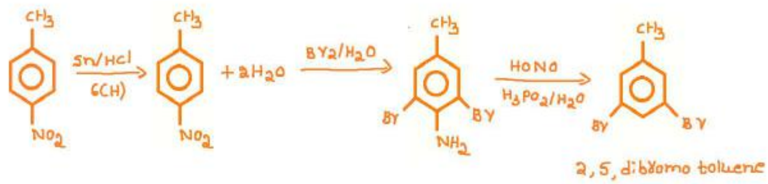

CH3
Br
No2

C)


CH3
PO2H
Br
No2

D)

SOL:

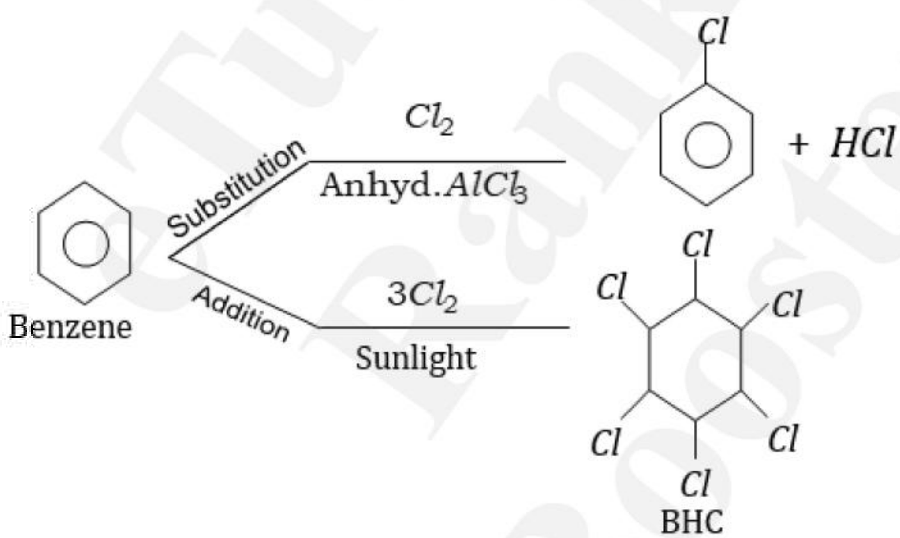


74. (i) Chlorobenzene and (ii) benzene hexachloride are obtained from benzene by the reaction of chlorine, in the presence of

A) (i) Direct sunlight and (ii) anhydrous $AlCl_3$ **B)** (i) Sodium hydroxide and (ii) sulphuric acid

C) (i) Ultraviolet light and (ii) anhydrous $FeCl_3$ **D)** (i) Anhydrous $AlCl_3$ and (ii) direct sunlight **E)**

SOL:



75. Which one of the following sets of quantum numbers represents an impossible arrangement?

A)

n	l	m	s
3	2	-2	1/2

B)

n	l	m	s
4	0	0	1/2

C)

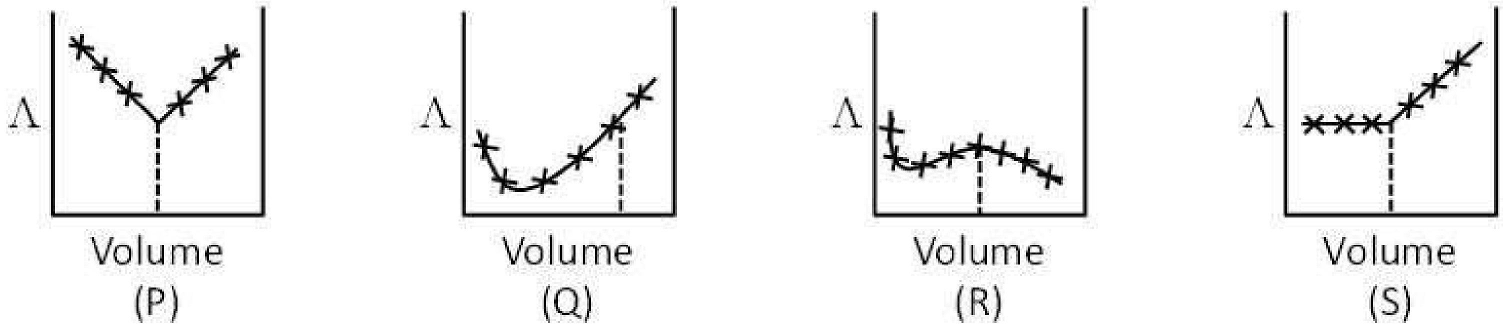
n	l	m	s
3	2	-3	1/2

D)

n	l	m	s
5	3	0	-1/2

SOL: Minimum value of m is - l

76. $AgNO_3$ (aq) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance ($\wedge$) versus the volume of $AgNO_3$ is



A) (P) **B)** (Q) **C)** (R) **D)** (S)

SOL: The plot of conductance (Λ) versus the volume of $AgNO_3$ is as given by (S). This is as explained below.

Initial conductance (Λ) of solution was due to $K^+_{(aq.)}$ and $Cl^-_{(aq.)}$

On addition of $AgNO_3$,

the reaction occurs as,

$AgNO_3$ (aq.)+KCl (aq.) $\rightarrow$ AgCl $\downarrow$ + KNO_3

$AgNO_3$ acts as limiting reagent up to complete precipitation. The conductance up to precipitation is a straight line parallel to x-axis due to replacement of Cl^-ions with NO_3^- ions (ionic mobility of NO_3^- and $Cl^-$are almost same). After complete precipitation, with further addition of $AgNO_3$, increase in the conductance is observed due to added Ag^+, NO_3^- ions.

77. A 5% solution of cane sugar (molar mass 342) is isotonic with 1% of a solution of an unknown solute. The molar mass of unknown solute in g.mol is

A) 136.2 **B)** 171.2 **C)** 68.4 **D)** 34.2

SOL: For isotomic solutions

$C_1 = C_2$

$\frac{5\times10}{342} = \frac{1\times10}{GMW}$

$\frac{5}{342} = \frac{1}{GMW}$

GMW = 68.4

78. F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution.

This is attributed to many factors except

A) Heat of dissociation **B)** Electron affinity **C)** Ionisational potential **D)** Heat of hydration

SOL: Due to high heat of hydration of F $^{\Theta}$ ion

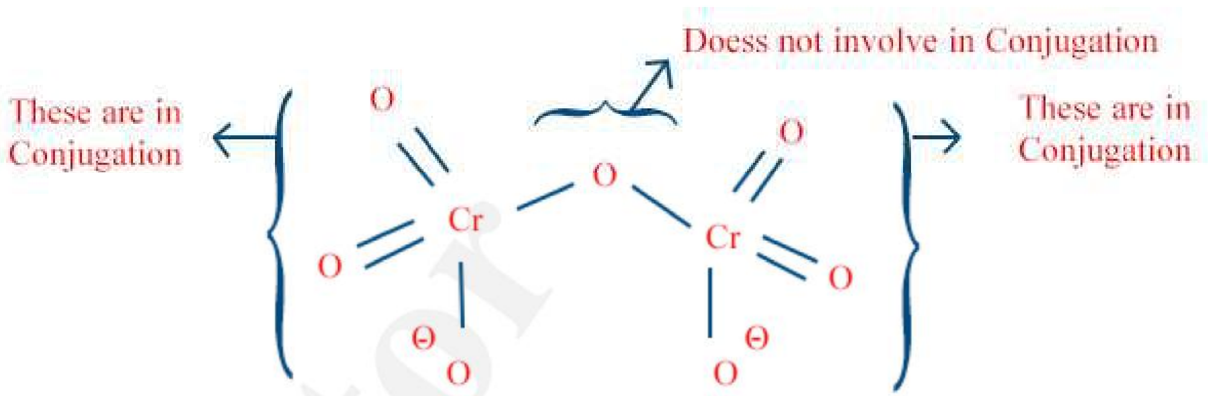
79. The bonds present in the structure of dichromate ion are

A) Four equivalent Cr - O bonds only **B)** Six equivalent $Cr - O$ bonds and one $O - O$ bond

C) Six equivalent $Cr - O$ bonds and one $Cr - Cr$ bond

D) Six equivalent $Cr - O$ bonds and one $Cr - O - Cr$ bond

SOL:



80. Compare vitamin List I with its deficiency disease List II.

List-I	**List-II**
A. Vitamin-B_{12}	1. Sterility
B. Vitamin-B_6	2. Hemorrhagic conditions
C. Vitamin-E	3. Pernicious anaemia
D. Vitamin-K	4. Skin disease

A)

A	B	C	D
1	2	3	4

B)

A	B	C	D
2	3	4	1

C)

A	B	C	D
3	4	1	2

D)

A	B	C	D
3	4	2	1

SOL:

Sl. No	**Name of Vitamins**	**Sources**	**Deficiency diseases**
1	Vitamin A	Fish liver oil, carrots, butter and milk	Xerophthalmia (hardening of cornea of eye) Night blindness
2	Vitamin B_1 (Thiamine)	Yeast, milk, green vegetables and cereals	Beri beri (loss of appetite, retarded growth)

3	Vitamin B_2 (Riboflavin)	Milk, egg white, liver, kidney	Cheilosis (fissuring at corners of mouth and lips), digestive disorders and burning sensation of the skin.
4	Vitamin B_6 (Pyridoxine)	Yeast, milk, egg yolk, cereals and grams	Convulsions
5	Vitamin B_{12}	Meat, fish, egg and curd	Pernicious anemia (RBC deficient in hemoglobin)
6	Vitamin C (Ascorbic acid)	Citrus fruits, amla and green leafy vegetables	Scurvy (bleeding gums)
7	Vitamin D	Exposure to sunlight, fish and egg yolk	Rickets (bone deformities in children) and osteoma Lacia (Soft bones and joint pain in adults)
8	Vitamin E	Vegetable oils like wheat germ oil, sunflower oil, etc.,	Increased fragility of RBCs and muscular weakness
8	Vitamin K	Green leafy vegetables	Increased blood clotting time

81. A Hydrated salt $CaCl_2$ X H_2O undergoes 49.32% loss in weight on heating and becomes anhydrous. The value of 'X' will be

SOL:

% loss in weight of compound $= \frac{18x}{111+18x} \times 100$

$= 49.32$

$18x \times 100 = 49.32\,(111+18x)$

$1800x = 5474.52 + 887.76x$

$1800x - 887.76x = 5474.52$

$912.24x = 5474.52$

$x = \frac{5474.52}{912.24}$

$= 6$

Formula of hydrated salt is $CaCl_2 \cdot 6H_2O$.

82. 1 mole of an ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ. The final temperature of the gas is equal to ________ K. ($C_v = 20$ J $mole^{-1}$ l^{-1})

SOL: $W = n\,C_v\,(T_2 - T_1)$

$-3000 = 1 \times 20\,(T_2 - 300)$

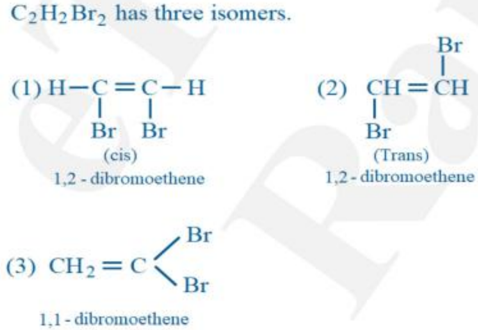
$-150 = T_2 - 300$

$T_2 = 150$ K

83. Number of isomers of molecular formula $C_2H_2Br_2$ are

SOL:

$C_2H_2Br_2$ has three isomers.



84. In a zero-order reaction, 47.5% of the reactant remains at the end of 2.5 hours. The amount of reactant consumed in one hour is _________ %.

SOL:

For zero order reaction

$$K = \frac{[R]_0 - [R]}{t}$$

Let $[R]_0 = 100$

At $t = 2.5$ hr

$[R] = 47.5$

$$\therefore K = \frac{100 - 47.5}{2.5}$$

$K = 21$

At $t = 1$ hr

$$21 = \frac{100 - [R]}{1}$$

$[R] = 100 - 21 = 79$

$\therefore$ At $= 1$ hr amount of substance consumed $= 100 - 79$

$= 21\%$

85. The possible numbers of isomers for the complex $[MCl_2\ Br_2]\ SO_4$ will be __________.

SOL:

The possible number of isomers for the complex compound $[MCl_2Br_2]SO_4$ is 2.

86. The solubility product (K_{sp}) of the sparingly soluble salt (MX) at $298\ K$ is 2.50×10^{-9}. The solubility of the salt at this temperature is $x \times 10^{-5}\ mol\ L^{-1}$. The value of x is______.

SOL:

$$MX_{(s)} \rightleftharpoons \underset{S}{M^+_{(aq)}} + \underset{S}{X^-_{(aq)}}$$

$$K_{sp} = [M^+][X^-]$$

$$\therefore \quad K_{sp} = S^2$$

$$\therefore \quad S^2 = 2.50 \times 10^{-9}$$

$$S = \sqrt{2.50 \times 10^{-9}}$$

$$= \sqrt{25.0 \times 10^{-10}}$$

$$= 5.00 \times 10^{-5}\ mol\ L^{-1}$$

$$\therefore \quad \text{Value of } x = 5$$

87. A sample of 2.5 moles of N_2H_4 loses 25 moles of electrons on being converted to a new compound X. Assuming that there is no loss of nitrogen in the formation of the new compound, what is the oxidation number of nitrogen in compound 'X'?

SOL: Per mole of N_2H_4 loss of electrons $= \frac{25}{2.5} = 10$

$\therefore$ increase in $O.S.$ of each N atom $= \frac{25}{2.5} = 5$

$\therefore$ O.S. of N. in $N_2H_4 = -2 + 5 = +3$

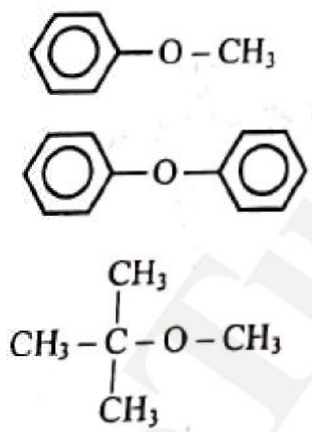
88. Among the following oxides, how many amphoteric? $CO_2, PbO_2, GeO_2,\ B_2O_2, Al_2O\ Tl_2O_3, Ga_2O_3, SiO_2, SnO_2$

SOL: In group 13, Al_2O_3 and Ga_2O_3 are amphoteric oxides.
In group 14, SnO_2 and PbO_2 are amphoteric oxides.

89. How many of the following can be prepared as a major product in williamson synthesis (Etherification)

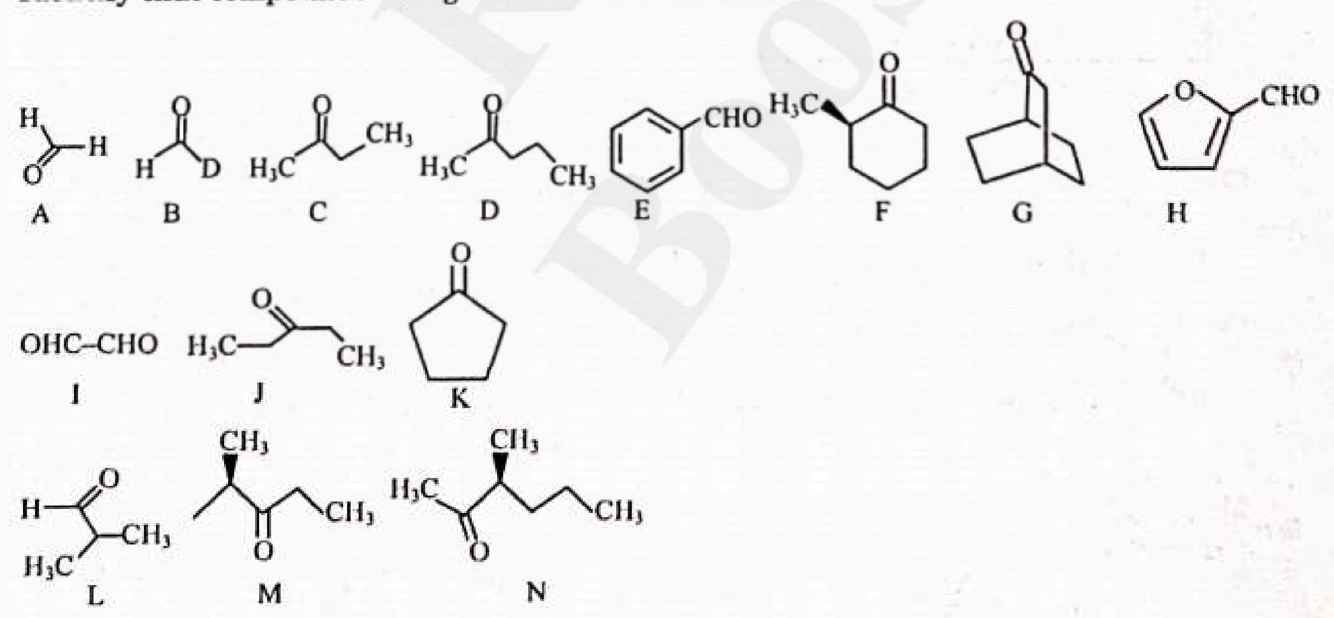
CH_3-O-CH_3

$CH_3-O-C_2H_5$



SOL: At least one 1° alkyl grp should be present in Williamson synthesis.

90. **Identify that compounds that give Cannizaro reaction**



SOL: Carbonyl compound without $\alpha - \mathbf{H}$ can able to gives Cannizaro reaction
$\mathbf{A}, \mathbf{B}, \mathbf{E}, \mathbf{H}, \mathbf{I}, \mathbf{L}$ gives Cannizaro reaction

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